۱) الف) $\tan\frac{11\pi}{4} + \sin\frac{10\pi}{4}\cos\frac{12\pi}{4} \Rightarrow \tan(2\pi+\frac{3\pi}{4}) + \sin(12\pi+\frac{3\pi}{4})\cos(12\pi+\frac{\pi}{4})$

$-1 - \frac{\sqrt{2}}{2}\times\frac{-\sqrt{2}}{2} \Rightarrow -1+\frac{1}{2} = \boxed{-\frac{1}{2}}$ ($\frac{1}{2}$)

ب) $\tan\frac{17\pi}{4}\sin\frac{11\pi}{3} + \cos\frac{10\pi}{3} \Rightarrow \tan(2\pi+\frac{\pi}{4})\times\sin(4\pi+\frac{\pi}{3}) + \cos(2\pi+\frac{4\pi}{3})$

$-\frac{\sqrt{2}}{2}\times\frac{-\sqrt{3}}{2} - \frac{1}{2} = \boxed{0}$

$+\frac{\sqrt{3}}{4} = +\frac{1}{2}$

$\Rightarrow \tan(\frac{3\pi}{4}) + \sin(\frac{7\pi}{6})\times\cos(\frac{6\pi}{3}) = -\frac{1}{2}$

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۲) $\cot x = 4$ ، $\frac{2\cos x - \sin x}{\cos x + \sin x}$

$\frac{\cos x}{\sin x} = 4 \Rightarrow \cos x = 4\sin x$ ← جایگذاری

$\frac{3\times(4\sin x) - \sin x}{4\sin x + \sin x} = \frac{11\sin x}{5\sin x} = \boxed{\frac{11}{5}}$

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۳) $\sin\alpha = 2\cos\alpha$ ، α در ربع سوم ، $\cos\alpha = ?$

$x^2+y^2=1 \longrightarrow \sin^2\alpha + \cos^2\alpha = 1$ ← جایگذاری $\Rightarrow (2\cos\alpha)^2 + \cos^2\alpha = 1^2 \Rightarrow 4\cos^2\alpha + \cos^2\alpha = 1$

$5\cos^2\alpha = 1 \Rightarrow \cos^2\alpha = \frac{1}{5} \rightarrow \cos\alpha = \pm\frac{1}{\sqrt{5}} \Rightarrow \cos\alpha = \pm\frac{\sqrt{5}}{5}$

چون در ناحیه سوم است ← $\boxed{-\frac{\sqrt{5}}{5}}$ جواب

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۴) الف) $\cos(\frac{11\pi}{4}+\alpha) \Rightarrow \cos(2\pi+\frac{3\pi}{4})\times\cos\alpha - (\sin(2\pi+\frac{3\pi}{4})\times\sin\alpha) \Rightarrow \frac{\sqrt{2}}{2}\times\frac{-\sqrt{2}}{10} - \frac{\sqrt{2}}{2}\times\frac{-\sqrt{2}}{10}$

$1-\sin^2\alpha = \cos^2\alpha \rightarrow 1-\frac{4}{100} = \cos^2\alpha \Rightarrow \pm\sqrt{\frac{96}{100}} = \cos^2\alpha \xrightarrow{\text{ناحیه ۲}} \boxed{-\frac{\sqrt{24}}{10}}$

$\frac{-12+2}{20} = \frac{-12}{20} = \boxed{-\frac{3}{5}}$ جواب

ب) $1+\tan^2\alpha = \frac{1}{\cos^2\alpha} \Rightarrow \frac{49}{64}+\frac{1}{64} = \frac{50}{64\alpha} \Rightarrow \cos^2\alpha = \pm\sqrt{\frac{64}{50}} \xrightarrow{\text{ناحیه ۱}} \cos\alpha = \frac{\sqrt{2}}{10} \Rightarrow \sin\alpha = \frac{\sqrt{98}}{10}$

$\sin(\frac{11\pi}{4}+\alpha) \Rightarrow \sin(2\pi+\frac{3\pi}{4})\times\cos\alpha + \sin\alpha\times\cos(2\pi+\frac{3\pi}{4}) \Rightarrow -\frac{\sqrt{2}}{2}\times\frac{\sqrt{2}}{10} + \frac{\sqrt{2}}{10}\times-\frac{\sqrt{2}}{2} =$

$\frac{-14-2}{20} = \frac{-16}{20} = \boxed{-\frac{4}{5}}$ جواب

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۵) $4\sin^2 x + \cos^2 x = \frac{5}{4}$ ، $\tan^2 x = ? \Rightarrow \frac{1}{\cos^2\alpha} - 1 \Rightarrow \frac{3}{2} - \frac{1}{2} = \boxed{\frac{1}{2}} \Rightarrow \tan^2 x$

$\sin^2\alpha = 1-\cos^2\alpha$

$2(1-\cos^2 x) + \cos^2 x = \frac{5}{4} \Rightarrow 2-\cos^2 x = \frac{5}{4} \Rightarrow \frac{8}{4}-\frac{5}{4} = \cos^2 x \Rightarrow \cos^2 x = \frac{1}{4} \Rightarrow \frac{1}{\cos^2 x} = \frac{4}{1}$

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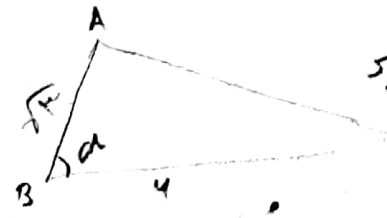
۶) ۲ برابر است جواب



$S_{ABC} \Rightarrow \frac{1}{2}\times 4\times\sqrt{2}\times\sin\alpha = \frac{8\sqrt{2}}{10} \Rightarrow 2\sqrt{2}\sin\alpha = \frac{8\sqrt{2}}{10}$

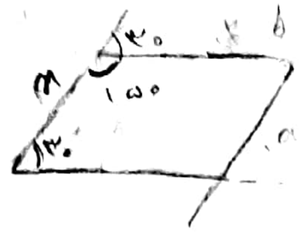
$\sin\alpha = \frac{4\sqrt{2}}{10}\times\frac{1}{2\sqrt{2}}$

$\sin\alpha = \frac{4}{10}\times\frac{\sqrt{2}}{2}$

$\sin\alpha = \frac{\sqrt{2}}{5}$ ← بیشترین مقدار $\alpha = 45°$ ← کمترین مقدار $\alpha = 0°$

$\frac{120}{60} = \boxed{2}$ جواب

۱۷) ضلع کوچیک = x

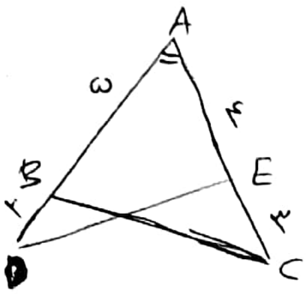


$\frac{a}{b} = \frac{2}{3} \Rightarrow b = \frac{3}{2}a \Rightarrow x = \frac{3x}{2}$

مساحت متوازی الاضلاع $= a \times b \times \sin 150^\circ \Rightarrow 54 = x \times (\frac{3x}{2}) \times \frac{1}{2} \Rightarrow 54 = \frac{3x^2}{4}$

$x^2 = 54 \times \frac{4}{3} \Rightarrow x^2 = 72 \Rightarrow x = 6\sqrt{2} = a \;,\; b = \frac{3}{2} \times 6\sqrt{2} \Rightarrow 9\sqrt{2}$

محیط متوازی الاضلاع = ۲× (مجموع ۲ ضلع مجاور) $\Rightarrow 2 \times (6\sqrt{2} + 9\sqrt{2}) = \boxed{30\sqrt{2}}$



$S_{ABC} - S_{ADE} = 1/75 \Rightarrow (5 \times 7 \times \frac{1}{2} \times \sin\hat{A}) - (7 \times 4 \times \frac{1}{2} \times \sin\hat{A})$

$\tan\hat{A} = ?$

$\frac{35}{2}\sin\hat{A} - \frac{28}{2}\sin\hat{A} = 1/75 \Rightarrow \frac{7}{2}\sin\hat{A} = 1/75$

$\sin\hat{A} = \frac{175}{100} \times \frac{2}{7}$

$\sin\hat{A} = \frac{1}{2} \Rightarrow \boxed{\hat{A} = 30^\circ}$ (چون حاده است)

$\tan\hat{A} = \boxed{\tan 30^\circ = \frac{\sqrt{3}}{3}}$

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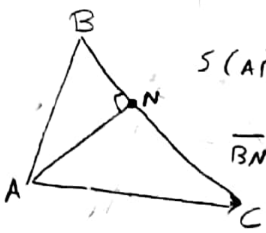
۶)

$2\overline{BN} = 3\overline{NC} \;,\; S_{ABC} = 3 S_{BMN} \qquad \frac{BM}{AM} = ?$



$S(ABN) = \frac{BN}{BC} \times S(ABC) = \frac{3}{5} \times 3 \Rightarrow \frac{9}{5} \Rightarrow \frac{S_{BMN}}{S_{ABC}} = \frac{BM}{BA} \times \frac{BN}{BC}$

ادامه در پایین صفحه

$\overline{BN} = \frac{3}{2}\overline{NC} \Rightarrow \overline{BC} = \frac{3}{2}\overline{NC} + \overline{NC} \Rightarrow \overline{BC} = \frac{5}{2}\overline{NC} \Rightarrow \frac{BN}{BC} = \frac{\frac{3}{2}NC}{\frac{5}{2}NC} = \frac{3}{5} \;,\; \frac{NC}{BC} = \frac{2}{5}$

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۱۰)

$\frac{1}{\sqrt{\cos^2\alpha}} - \tan\alpha = \frac{1+\sin\alpha}{|\cos\alpha|} \Rightarrow \frac{1-1-\sin\alpha}{|\cos\alpha|} = \frac{\sin\alpha}{\cos\alpha} \Rightarrow \frac{-\sin\alpha}{|\cos\alpha|} = \frac{\sin\alpha}{\cos\alpha}$

$\sin < 0$
$\cos > 0$ یا $\cos < 0$

$\cos\alpha > 0 \longrightarrow$ ناحیه ۳ و چهار

$\frac{|\sin\alpha|}{\cos\alpha} = -\frac{1}{\cot\alpha}$ ($\sin\alpha > 0$)

$\frac{|\sin\alpha|}{\cos\alpha} = \frac{-\sin\alpha}{\cos\alpha} \Rightarrow \sin\alpha > 0 \;,\; \cos < 0 \Rightarrow$ ناحیه ۲ و ۳

} ⇐ اشتراک ⇐ ناحیه ۳ مثلثاتی

ادامه سوال ۹ ⇐

$\frac{S_{BMN}}{S_{ABC}} = \frac{BM}{BA} \times \frac{BN}{BC} \Rightarrow \frac{1}{3} = \frac{BM}{BA} \times \frac{3}{5} \Rightarrow \frac{BM}{BA} = \frac{5}{9}$

$BA = BM + AM \Rightarrow AM = BA - BM = BA(1 - \frac{BM}{BA}) = 4 \Rightarrow \frac{BM}{AM} = \frac{\frac{5}{9}BA}{\frac{4}{9}BA} = \frac{5}{4}$

$\boxed{\frac{BM}{AM} = \frac{5}{4}}$

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