Subject Date

کوئیز مدرسه

(۱)

$\{1\},\ \{2,3,4\},\ \{5,6,7,8,9\},\ \dots$

$8^2=64,\ 9^2=81 \Rightarrow \{65,66,67,68,69,\dots,81\}$

۱۷ تا

$\frac{65+81}{2}=\boxed{73}$

(۲)

جمله n ام a_n ، جمله n ام $b_n \longrightarrow b_n = 3a_n$

دسته اول $\longrightarrow \{1,2,3\}$ دسته دوم $\longrightarrow \{4,5,\dots,12\}$

دسته سوم $\longrightarrow \{13,\dots,39\}$ دسته چهارم $\longrightarrow \{40,\dots,120\}$

دسته پنجم $\longrightarrow \{121,\dots,363\}$ $\frac{\overset{484}{121+363}}{2}=\boxed{242}$

(۳) ?

$a_2+a_5+a_8+\dots+a_{29}$

$n=3k+2$

$a=-1 \quad a_n=\left[\frac{n}{k+2}\right]+a \Rightarrow a_n=\left[\frac{n}{k+2}\right]$

$\longrightarrow 2,5,8,11,14,17,20,23,26,29$

$k=0,\ k=1,\ k=2,\ k=3,\ k=4,\ k=5,\ k=6,\ k=7,\ k=8,\ k=9$

$1+0+1+1+1+1+1+1+1+1=\boxed{8}$

ادامه $a_1=\frac{1}{5}(1)^2-\frac{4}{5}(1)+13 \Rightarrow a_1=\frac{62}{5}=\frac{124}{10}=12.4$

$a_{15}=\frac{1}{5}(15^2)-\frac{4}{5}(15)+13 \Rightarrow a_{15}=46$

$\Rightarrow \frac{46}{12.4}=\frac{115}{31}=3.7$

(۴)

$a_n=an^2+bn+c,\quad a_5=14$

$a_4 \to a=\frac{1}{70}\times \overset{14}{a}=\frac{1}{5}$ $\quad \frac{1}{5}(5^2)+b(5)+c=14$ $\boxed{5b+c=9}$

$a_7=17.2 \longrightarrow \frac{1}{5}(7^2)+b(7)+c=17.2 \Rightarrow \boxed{7b+c=7.4}$

$\Rightarrow 7b+(9-5b)=7.4 \Rightarrow 2b+9=7.4 \Rightarrow \boxed{b=-\frac{4}{5}} \Rightarrow c=9-5b$

$\Rightarrow c=9-5\left(-\frac{4}{5}\right)=9+4=13$

$L_{10} = 0 \Rightarrow 10a + b = 0 \Rightarrow b = -10a$

$L_n = an + b \qquad L_n = an - 10a = a(n-10)$ (۵)

گوهر کامرانی

$A_4 = L_2 \Rightarrow a(2-10) = -8a$

$A_8 = L_7 \Rightarrow a(7-10) = -3a$

$A_8 - A_4 = (8-4)d = 4d$

$-3a - (-8a) = 4d$

$\Rightarrow 5a = 4d \Rightarrow d = \frac{5}{4}a \longrightarrow L_{15} = a(15-10) = 5a$

$\Rightarrow \frac{L_{15}}{d} = \frac{5a}{\frac{5}{4}a} = \boxed{4}$

(۶)

$6(a_1+d)^2 = 5(a_1+2d)a_1 + 3(a_1+d)a_1$

$6a_1^2 + 12a_1 d + 6d^2 = 8a_1^2 + 13a_1 d$

$\Rightarrow \boxed{2a_1^2 + a_1 d - 6d^2 = 0}$

$\frac{a_4}{d} = \frac{a_1+3d}{d} = \frac{a_1}{d} + 3$

$\boxed{d \neq 0} \qquad 2\frac{a_1^2}{d^2} + \frac{a_1 d}{d^2} - 6\frac{d^2}{d^2} = 0$

$\Rightarrow 2\left(\frac{a_1}{d}\right)^2 + \left(\frac{a_1}{d}\right) - 6 = 0$

$\Delta = 1^2 - 4\times 2\times(-6) = 1 + 48 = \boxed{49} \Rightarrow \sqrt{49} = 7$ ۲ ریشه $\rightarrow \frac{-1\pm 7}{4}$

$\rightarrow \frac{-1+7}{4} = \frac{3}{2} = x_1 \quad , \quad \frac{-1-7}{4} = -2 = x_2$

$\left(\frac{a_4}{d}\right)_1 = x_1 + 3 = \frac{3}{2} + 3 = \boxed{\frac{9}{2}}$

$\left(\frac{a_4}{d}\right)_2 = x_2 + 3 = \boxed{1}$

کوثر علامتی

7

$$2b = a + c$$

$$\left(\frac{a}{2}\right)^2 = \left(\frac{c}{4}\right) \times b \Rightarrow \frac{a^2}{4} = \frac{cb}{4} \Rightarrow a^2 = cb$$

$$q = \frac{\frac{a}{2}}{\frac{c}{4}} = \frac{a}{2} \times \frac{4}{c} = \frac{2a}{c} \quad , \quad q = \frac{b}{\frac{a}{2}} = \frac{2b}{a}$$

$$\Rightarrow \frac{2a}{c} = \frac{2b}{a} \Rightarrow \frac{a}{c} = \frac{b}{a}$$

$$c = 2b - a \xrightarrow{\text{جایگذاری}} a^2 = (2b - a)b$$

$$\Rightarrow a^2 = 2b^2 - ab \Rightarrow a^2 + ab - 2b^2 = 0$$

$$b \neq 0 \xrightarrow{\text{تقسیم}} \left(\frac{a}{b}\right)^2 + \left(\frac{a}{b}\right) - 2 = 0 \Rightarrow x^2 + x - 2 = 0$$

$$(x+2)(x-1) = 0 \quad \begin{cases} \frac{a}{b} = -2 \Rightarrow a = -2b \\ \frac{a}{b} = 1 \Rightarrow a = b \end{cases}$$

$$a = -2b \qquad 2b = c + (-2b) \Rightarrow c = 4b$$

$$q = \frac{2a}{c} \quad . \quad q = \frac{-4b}{4b} = -1$$

$$2q = 2 \times (-1) = -2$$

$$a = -2b \ , \ c = 4b$$

$$\left(\frac{a}{b}\right)^2 + 2\left(\frac{a}{b}\right) - 4 = 0 \Rightarrow x^2 + 2x - 4 = 0$$

$$x = \frac{-2 \pm \sqrt{2^2 + 16}}{2} = -1 \pm \sqrt{5}$$

$$\frac{a}{b} = -1 + \sqrt{5} \qquad \frac{a}{b} = -1 - \sqrt{5}$$

$$q = \frac{a}{2b - a} = \frac{\frac{a}{b}}{2 - \frac{a}{b}}$$

کریم علایی

8

$b = ar \quad , c = ar^2$

$2(2a) = c + 3b \Rightarrow 4a = c + 3b$

$4a = ar^2 + 3(ar)$

$4 = r^2 + 3r \rightarrow r^2 + 3r - 4 = 0 \Rightarrow (r+4)(r-1) = 0$

$\cancel{r=1} \quad r = -4$

$\dfrac{T_8}{T_5} = r^3 = (-4)^3 = \underline{-64}$?

9

$\dfrac{a_1 r^5}{(a_1 r)^3} + \dfrac{a_1 r}{a_1^2} = 2$

$\dfrac{a_1 r^5}{a_1^3 r^3} + \dfrac{r}{a_1} = 2 \Rightarrow \dfrac{r^2}{a_1^2} + \dfrac{r}{a_1} = 2 \Rightarrow r^2 + r a_1 = 2a_1^2$

$2a_1^2 - r a_1 - r^2 = 0$

$a_1 = \dfrac{r \pm \sqrt{9r^2}}{4} \rightarrow |r| = r \rightarrow r > 0 \;,\; a_1 = -\dfrac{r}{2}$

$\rightarrow |r| = -r \rightarrow r < 0 \;,\; a_1 = r$

$\dfrac{a_1}{r} = \dfrac{r}{r} = 1 \qquad \dfrac{a_1}{r} = -\dfrac{1}{2}$ ✓

10

$a_3 = \sqrt{a_4} \;,\; a_5 = 27$

$a_3 = a_1 r^2 \;,\; a_4 = a_1 r^3 \;,\; a_5 = a_1 r^4$

$a_1 r^2 = \sqrt{a_1 r^3} \Rightarrow a_1^2 r^4 = a_1 r^3$

$a_5 = 27 \Rightarrow a_1 r^4 = 27 \qquad (a_1 r) r^3 = 27 \Rightarrow 1 \cdot r^3 = 27$

$a_1 \cdot r = 1 \qquad a_1 \cdot 3 = 1 \Rightarrow a_1 = \dfrac{1}{3} \qquad \dfrac{r^3 = 27}{r = 3}$

$\dfrac{1}{2} - \dfrac{1}{3} = \boxed{\dfrac{1}{6}}$