نام : سنا فاضلی   دهم A دختر   تکلیف : ۱۷

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$\overset{1}{\{1\}},\ \overset{2}{\{2,3,4\}},\ \overset{3}{\{5,6,7,8,9\}}$

$(n-1)^2+1$: عدد اول n^2 : عدد آخر

$9 : \{65, \dots, 81\} \rightarrow \frac{81+65}{2} = \frac{146}{2} = 73 \rightarrow$ واسطه حسابی

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$\{1,2,3\}, \{4, \dots, 12\}, \{13, \dots, 39\}, \{40, \dots, 120\}, \{121, \dots, 363\}$

میانگین $= \frac{(363+121)}{2} = 242$

۳-

$$a_n = \begin{cases} 2^k & ; n=3k \\ -2k+4 & ; n=3k+1 \\ \left[\frac{n}{k+2}\right]+a & ; n=3k+2 \end{cases}$$

$n=0,3,6,9,\dots$ $a_0=1$ $a_3=2$ $a_6=4$ $a_9=8$

$n=1,4,7,\dots$ $a_1=4$ $a_4=2$ $a_7=0$

$n=2,5,8,\dots,29$ $a_2=1+a$ $a_5=1+a$ $a_8=2+a$

$a_0+a_1+a_2+\dots+a_9 = 19 = 25+3a \rightarrow a=-2$

$a_2+a_5+a_8+\dots+a_{29} = 10a + \{1+1+2+2+\dots+2\} = 10a+18 = -20+18 = -2$

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$$\left.\begin{array}{l} a_5 = 14 \rightarrow (25a+5b+c=14)\times -1 \\ a_7 = 17.2 \rightarrow 49a+7b+c=17.2 \end{array}\right\} \Rightarrow 24a+2b=3.2 \rightarrow 12a+b=1.6$$

$a = \frac{1}{70}\times(-14) = -0.2 \longrightarrow 24\times -0.2+b=1.6 \rightarrow b=4 \longrightarrow -5+20+c=14 \rightarrow c=11$

$a_{15} = 225(-0.2)+15(4)+(-1) = 14$

$a_1 = -0.2+4+(-1) = 2.8$ $\frac{14}{2.8} = 5$

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فرض می کنیم دنباله حسابی $\rightarrow t_n = n-10$ باشد $\rightarrow t_{15} = 5$

$a_4 = 2-10 = -8$

$a_8 = 7-10 = -3$ $d = \frac{-3-(-8)}{8-4} = \frac{5}{4}$ $\longrightarrow \frac{t_{15}}{d} = \frac{5}{\frac{5}{4}} = 4$

۶-

فرض می کنیم : $d=1 \longrightarrow 6(a+1)^2 = 5(a+2)a+3(a+1)a = 6a^2+12a+6 = 5a^2+10a+3a^2+3a$

$\rightarrow 2a^2+a-6=0 \rightarrow \Delta = b^2-4ac = 1+48 = 49$ $a = \frac{-\cancel{b}^{\,-1} \pm \sqrt{49}}{\cancel{2a}_{\,4}} = -2, 1.5$

اگر $a=-2 \rightarrow a_4=1 \quad : (-2+3=1)$

اگر $a=1.5 \rightarrow a_4=4.5 \quad : (1.5+3=4.5)$ $\longrightarrow$ (d یک فرض شده بود پس $\frac{a_4}{d}$ همان جملات چهارم هستند)

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$a+c=2b$

$\frac{1}{4}bc = \frac{1}{4}a^2 \rightarrow bc = a^2 \rightarrow a^2 = (a+d)(a+2d) = a^2+3ad+2d^2 \rightarrow 3ad+2d^2=0 \rightarrow 3a+2d=0 \rightarrow d=-\frac{3}{2}a$

$2q = 2\left(\frac{\frac{1}{2}a}{b}\right) = \frac{a}{b} = \frac{a}{a+d} = \frac{a}{a-\frac{3}{2}a} = \frac{a}{-\frac{1}{2}a} = -2$

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$b^2=ac \to a=\frac{b^2}{c} \longrightarrow \underbrace{2(2a)=3b+c}_{\text{واسطه حسابی}} \to 4a=3b+c$

$\longrightarrow 4\left(\frac{b^2}{c}\right)=3b+c \xrightarrow{\times c} 4b^2=3bc+c^2 \to c^2+3bc-4b^2=0 \xrightarrow{\div b^2} \left(\frac{c}{b}\right)^2+3\left(\frac{c}{b}\right)-4=0$

$\xrightarrow{\frac{c}{b}=x} x^2+3x-4=0 \xrightarrow{\text{تجزیه}} (x+4)(x-1)=0 \to x=-4$, $x=1 \times \to$ دنباله ثابت

$\frac{c}{b}=-4 \to \frac{b}{c}=-\frac{1}{4} \to \frac{ar^8}{ar^5}=r^3=\left(-\frac{1}{4}\right)^3=-\frac{1}{64}$

۹-

$\frac{a_6}{(a_2)^3}+\frac{a_2}{(a_1)^2}=2 \to \frac{a_1r^5}{(a_1r)^3}+\frac{a_1r}{a_1^2}=2 \to \frac{a_1r^5}{a_1^3r^3}+\frac{r}{a_1}=2 \to \frac{r^2}{a_1^2}+\frac{r}{a_1}=2 \xrightarrow{\times a_1^2} r^2+ra_1=2a_1^2$

$a=1$: فرض $\longrightarrow r^2+r(1)=2(1)^2 \to r^2+r-2=0 \to (r-1)(r+2)=0 \to r=1$, $r=-2$

$\frac{a_1^2}{a_1r}=\frac{a_1}{r} \to \frac{1}{1}=1$, $-\frac{1}{2}$

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$t_3=\sqrt{t_4}$, $t_5=27$

$\underbrace{a}_{t_1}, \underbrace{ar}_{t_2}, \underbrace{ar^2}_{t_3}$

$ar^2=\sqrt{ar^3} \xrightarrow{\text{به توان ۲}} a^2r^4=ar^3 \to ar=1 \to$ جمله دوم

$ar^4=27 \to arr^3=27 \to r^3=27 \to r=3$

$ar=1 \xrightarrow{r=3} a=\frac{1}{3} \to$ جمله اول $\longrightarrow \frac{1}{2}-\frac{1}{3}=\frac{1}{6}$