۱)

دسته اول $\{1\}$ $\quad 1=1^2$

دسته دوم $\{2,3,4\}$ $\quad 4=2^2$

دسته سوم $\{5,6,7,8,9\}$ $\quad 9=3^2$

$\vdots$

دسته هشتم $\to 8^2=64$

دسته نهم $\to 9^2=81$

دسته نهم $\to \{65,66,\dots,81\}$

$$\frac{a+b}{2}=\frac{65+81}{2}=\frac{146}{2}=73$$

۲)

$b_n=a_{n+1}-1$

$a_n=\frac{1}{3}b_n \to b_n=3a_n$ $\quad \begin{cases} 3a_n=a_{n+1}-1 \\ a_{n+1}=3a_n+1 \end{cases}$

$a_1=1$

$b_1=3a_1=3\times1=3 \xrightarrow{\text{دسته اول}} \{1,2,3\}$

$n=2: \; a_2=3a_1+1=3\times1+1=4 \to \{4,5,\dots 12\}$

$n=3: \; a_3=3a_2+1=3\times4+1=13 \xrightarrow[\text{بزرگترین عضو}]{} b_3=3a_3=39$

$n=4 \; =3a_3+1=3\times13+1=39+1=40 \xrightarrow[\text{بزرگترین}]{} b_4=3\times40=120$

$n=5 \; =3a_4+1=3\times40+1=121 \xrightarrow[\text{بزرگترین}]{} 3\times121=363$

$\{121,122,\dots,363\}$

$$\frac{a_5+b_5}{2}=\frac{121+363}{2}=\frac{484}{2}=242$$

۳)

$a_0=2^0=1$

$a_1=-2\times0+4=4$

$a_2=\left[\frac{2}{0+2}\right]+a=1+a$

$a_3=2^1=2$

$a_4=-2\times1+4=2$

$a_5=\left[\frac{5}{1+2}\right]+a=1+a$

$a_6=2^2=4$

$a_7=-2\times2+4=0$

$a_8=\left[\frac{8}{2+2}\right]+a=2+a$

$a_9=2^3=8$

$S_{10}=25+3a=19$

$a=-2$

$a_2+a_5+a_8+\dots+a_{29}=$

$(-1)+0+\dots+0=-2$

۴)

$a_5=14 \qquad a_7=17.2$

$A=\frac{1}{70}(-a_5)=\frac{1}{70}\times(-14)=-\frac{14}{70}=-\frac{1}{5} \qquad A=-0.2$

$a_5=A(5^2)+B(5)+C=14$

$-0.2\times25+B\times5+C=14$

$5B+C=19 \quad \mathrm{I}$

$a_7=A(7^2)+B(7)+C=17.2$

$-0.2\times49+7B+C=17.2$

$7B+C=27 \quad \mathrm{II}$

$\begin{cases} 7B+\cancel{C}=27 \\ 5B+\cancel{C}=19 \end{cases}$

$2B=8 \to B=4 \qquad 5\times4+C=19$

$C=-1$

$a_n=-0.2n^2+4n-1$

$a_1=-0.2+4-1=2.8$

$a_{15}=-0.2\times225+60-1=14$

$$\frac{a_{15}}{a_1}=\frac{14}{2.8}=5$$

۵)

$a_4=b_2 \qquad a_8=b_7 \qquad b_{10}=0$

$b_n=b_m+(n-m)d', \quad a_n=a_m+(n-m)d$

$b_{10}=b_7+(10-7)d'=0 \qquad b_7+3d'=0$

$b_7=-3d' \to a_8=-3d'$

$a_8=a_4+(8-4)d \to a_8=a_4+4d \to a_4=a_8-4d$

$b_7=b_2+(7-2)d' \to b_7=b_2+5d' \qquad b_2=b_7-5d'$

$\xrightarrow[b_7=a_8,\; b_2=a_4 \text{ طبق فرض}]{} a_4=a_8-5d'$

$a_8-4d=a_8-5d' \qquad -4d=-5d' \to d'=\frac{4}{5}d$

$b_{15}=b_{10}+(15-10)d' \to b_{15}=0+5d'$

$b_{15}=5d' \to b_{15}=\cancel{5}\left(\frac{4}{\cancel{5}}d\right)=4d$

$$\frac{b_{15}}{d}=\frac{4d}{d}=4$$

(6

$a_2 = a + d$

$a_3 = a + 2d$

$6(a+d)^2 = 5(a+2d)a + 3(a+d)a$

$6(a^2+2ad+d^2) = (5a^2+10ad) + (3a^2+3ad)$

$6a^2+12ad+6d^2 = 8a^2+13ad$

$2a^2+ad-6d^2=0$

$2\left(\frac{a}{d}\right)^2+\left(\frac{a}{d}\right)-6=0$

$2x^2+x-6=0$

$2x^2+4x-3x-6=0$

$2x(x+2)-3(x+2)=0$

$(2x-3)(x+2)=0$

$2x-3=0 \rightarrow 2x=3 \quad x=\frac{3}{2}$

$x+2=0 \rightarrow x=-2$

$\frac{a}{d}=\frac{3}{2}$ یا $\frac{a}{d}=-2$

$\rightarrow -2+3=1$

$\rightarrow \frac{3}{2}+3=\frac{9}{2}$

$\frac{a_4}{d}=\frac{a+3d}{d}$

$\frac{a_4}{d}=\frac{a}{d}+3$

$\frac{a}{d}\in\left\{\frac{9}{2},1\right\}$

(7

متوالی حسابی $\rightarrow 2b=a+c$

هندسی $\rightarrow a^2=bc$

$a^2=(a+d)(a+2d)$

$=a^2+2ad+ad+2d^2$

$=\underbrace{a^2+3ad+2d^2}$

$3ad+2d^2=0$

$d(3a+2d)=0$

$3a+2d=0$

$2d=-3a$

$d=-\frac{3}{2}a$ I

$r=\frac{\frac{a}{2}}{b}$

$b=a+\left(-\frac{3}{2}a\right)=$

$=a-\frac{3}{2}a=-\frac{1}{2}a$

$b=-\frac{a}{2}$ II

$r=\frac{\frac{a}{2}}{b}=\frac{\frac{a}{2}}{-\frac{a}{2}}=-1$

$b=-a/2$

$c=a+2d=a+2\left(-\frac{3}{2}a\right)=a-3a=-2a$

متمایز $\rightarrow d\neq 0$

$=2r=2(-1)=-2$

(8

$\left.\begin{array}{l} b=a\times r \\ c=a\times r^2 \end{array}\right\} \longrightarrow 4a=3ar+ar^2$

$b^2=ac$

$2a=\frac{3b+c}{2} \rightarrow 4a=3b+c$ I

$4a=3ar+ar^2$

$\xrightarrow{\div a} 4=3r+r^2$

$r^2+3r-4=0 \longrightarrow (r+4)(r-1)=0 \longrightarrow r=-4$

$r=-4 \quad r=1$ غ ق ق

$\frac{a_9}{a_6}=r^{9-6}=r^3=(-4)^3=-64$

(9

$a_2=a\times r^{2-1}=ar$

$a_6=a\times r^{6-1}=ar^5$

$\frac{a_6}{(a_2)^3}+\frac{a_2}{a^2}=2 \rightarrow \frac{ar^5}{(ar)^3}+\frac{ar}{a^2}=2 \rightarrow$

جمله اول: $\frac{ar^5}{a^3r^3}=\frac{r^{5-3}}{a^{3-1}}=\frac{r^2}{a^2}$

جمله دوم $\frac{ar}{a^2}=\frac{r}{a}$

$\frac{r^2}{a^2}+\frac{r}{a}=2$

$\left(\frac{r}{a}\right)^2+\frac{r}{a}=2 \rightarrow \left(\frac{r}{a}\right)^2+\frac{r}{a}-2=0 \rightarrow \left(\frac{r}{a}+2\right)\left(\frac{r}{a}-1\right)=0 \longrightarrow \frac{r}{a}=-2$ یا $\frac{r}{a}=1$

$\frac{a^2}{a_2}=\frac{a^2}{ar}=\frac{a}{r} \longrightarrow \frac{a}{r}=\frac{1}{-2}=-\frac{1}{2}$ یا $\frac{a}{r}=\frac{1}{1}=1$

$\frac{a^2}{a_2}\in\left\{-\frac{1}{2},1\right\}$

(10

$a_3=\sqrt{a_4} \rightarrow a_3^{\,2}=a_4$

$a_3^{\,2}=a_3\times r$

$a_3=r$

$a_4=r\times r=r^2$

$a_5=r^2\times r=r^3$

$r^3=27 \quad r=3$

$a_5=a_1\times 3^4$

$27=a_1\times 81$

$a_1=\frac{27}{81}=\frac{1}{3}$

$\frac{1}{2}-a_1=\frac{1}{2}-\frac{1}{3}=\frac{3-2}{6}=\frac{1}{6}$