دهم ریاضی A

عارفه سعادت نیا

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$\{1\}, \{2,3,4\}, \{5,6,7,8,9\}, \{\ldots$

رقم آخر دسته نهم ← $n^2 \to 9^2 = 81$

رقم اول ← $(n-1)^2+1$ ← دسته دوم: $(2-1)^2+1=2$

دسته نهم $(9-1)^2+1 = 65$

واسطه حسابی $\frac{65+81}{2} = 73$

۲

$\{1,2,3\}, \{4,5,\ldots,12\}, \{13,14,15,\ldots,39\}, \{40,\ldots,120\}, \{121,\ldots,363\}$

دسته $5^{\text{ام}}$

$\frac{121+363}{2} = \frac{484}{2} = 242$

۳

$2^k \to n = 9,6,3,0 \to k = 0,1,2,\ldots$ $\qquad a_0 = 2^0 = 1$

$-2k+4, n = 1,4,7 \to k = 0,1,2,\ldots$ $\qquad a_1 = -2(0)+4 = 4$

$\left[\frac{n}{k+2}\right] + a \quad n = 2,5,8 \to k = 0,1,2,\ldots$ $\qquad a_2 = \left[\frac{2}{0+2}\right] + a = 1+a$

$a_3 = 2 \qquad a_4 = 2 \qquad a_5 = -2+1+a$

$a_6 = 4 \qquad a_7 = 0 \qquad a_8 = \frac{2}{5}+a = 2+a \qquad a_9 = 8$

$S_{10} = 1+4+1+a+2+2+1+a+4+0+2+a+8 = 3a = -6$

$a = -2$

$a_2 + a_5 + a_8 + \cdots + a_{29} = -2$

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$a = \frac{1}{70} \times -14 = -0.2$

$a_5 = a(5)^2 + 5b + c = 14$

$a_7 = a(7)^2 + 7b + c = 17.2$

$$\begin{cases} -25a - 5b - c = -14 \\ 49a + 7b + c = 17.2 \end{cases}$$

$24a + 2b = 3.2 \to b = 4$

$c \to 25(-0.2) + 5(4) + c = 14$

$c = -1$

$a_n = -0.2n^2 + 4n - 1$

$a_1 = 2.8$

$a_{15} = -0.2(15)^2 + 4(15) - 1 = 14$

$\frac{a_{15}}{a_1} = \frac{14}{2.8} = 5$

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$a_4 = a + 3d \to a' + d' = -8d'$

$a_8 = a + 7d \to a' + 6d' = -2d'$

$\to 4d = 5d'$

$d = \frac{5}{4}d'$

$a' + 9d' = 0 \to a' = -9d'$

$a_{15} = \frac{a' + 14d'}{d} = \frac{5d'}{\frac{5}{4}d'} = \frac{\frac{5}{1}}{\frac{5}{4}}$

۴ برابر

۶

$6a_2^2 = 5a_2a + 3a_2a$

$6a_2^2 - 3a_2a + 5a_2a \to 3a_2(\underbrace{2a_2 - a}_{a+2d}) = \underbrace{5a_2a}_{5(a_1+2d)a}$

$\frac{a_1}{d} = \frac{a_1+3d}{d}$

① $= \frac{-2d+3d}{d} = \frac{d}{d} = 1$

② $\frac{\frac{3}{2}d + 3d}{d} = \frac{\frac{9}{2}\cancel{d}}{\cancel{d}} = \frac{9}{2} = 4.5$

$3a_2(a_1+2d) = 5a_1(a_1+2d) \to 3a_2 = 5a_1$

$\underbrace{3(a_1+d)}_{3a_1+3d} = 5a_1$

$3d = 2a_1$

$a_1 = \frac{3}{2}d$

$a_1 + 2d = 0 \to a_1 = -2d$

7)

حسابی $a_n = c, b, a \to a = c+2d \quad b = c+d$

هندسی $b_n = \frac{c}{4}, \frac{a}{2}, b \to \frac{a}{2} = \frac{c}{4}(q), \; b = \frac{c}{4}(q^2)$

$b \to c+d = \frac{c}{4}q^2 \to d = \frac{c}{4}q^2 - c \to c\left(\frac{q^2}{4}-1\right)$

$a \to c+2d = \frac{c}{2}q \to 2d = \frac{c}{2}q - c \to c\left(\frac{q}{2}-1\right)$

$\frac{2d}{d} = \frac{c\left(\frac{q}{2}-1\right)}{c\left(\frac{q^2}{4}-1\right)} = \frac{\left(\frac{q}{2}-1\right)}{\left(\frac{q}{2}+1\right)\left(\frac{q}{2}-1\right)} \Rightarrow 2 = \frac{1}{\frac{q}{2}+1} \to q+2=1 \quad q=-1$

$2d = -2$

8)

$c, 2a, 3b \to 2a = c+d, \; 3b = c+2d$

$3b \to 3cq = c+2d \to 3cq - c = 2d \to c(3q-1) = 2d$

$2a \to 2cq^2 = c+d \to 2cq^2 - c = d \to c(2q^2-1) = d$

$\frac{2d}{d} = \frac{c(3q-1)}{c(2q^2-1)} \to 2 = \frac{3q-1}{2q^2-1}$

$4q^2 - 2 = 3q - 1 \to 4q^2 - 3q - 1 = 0 \to$

$(q-4)(q+1) = 0$

$q^2 - 3q - 4 = 0 \to q = \frac{4}{4} = 1$

$q = \boxed{-\frac{1}{3}}$

$\frac{a_9}{a_6} = q^3 = \frac{-1}{64}$

9)

$\frac{a_6}{(a_2)^3} + \frac{a_2}{a^2} = 2 \longrightarrow \frac{\cancel{a_1} q^5}{a_1^3 \cancel{q}} + \frac{\cancel{a_1} q}{a_1^2} = 2 \longrightarrow \frac{q^2}{a_1^2} + \frac{q}{a_1} = 2 \to \frac{q^2}{a_1^2} + 1\frac{q}{a_1} - 2 = 0$

$\left(\frac{q}{a_1}+2\right)\left(\frac{q}{a_1}-1\right) = 0$

$\frac{q}{a_1} = -2, \; 1$

$\frac{a_1^2}{a_2} = \frac{a}{q} \to \frac{a}{q} = 1 \, / \, -\frac{1}{2}$

10)

$a_3^2 = a_4 \to a_4 = a_3 q \to q = a_3$

$a_5 = 27$

$\frac{a_5}{a_3} = q^2 = \frac{27}{\frac{a_4}{q}} \to q^3 = 27 \to q = 3$

$a_5 = a_1 q^4 \to a_1 = \frac{a_5}{q^4} = \frac{27}{3^4} = \frac{27}{81} = \frac{1}{3}$

$\frac{1}{2} - \frac{1}{3} = \frac{1}{6}$