

$$\{1\}, \{2, 3, 4\}, \{5, 6, 7, 8\}, \dots \rightarrow \{(n-1)^2 + 1, (n-1)^2 + 2, \dots, n^2\} \quad (1)$$

$$\rightarrow n=9 \rightarrow \{76, 77, \dots, 81\} \rightarrow \text{دفعه مابقی} = \frac{81+76}{2} = \boxed{78.5}$$

$$\{1, 2, 3\}, \{4, 5, \dots, 12\}, \{13, \dots, 29\}, \{30, \dots, 45\}, \{46, \dots, 73\} \quad (2)$$

$$\text{مجموع} = 73 - 12 + 1 = 62 \rightarrow \text{میانگین} = \frac{\frac{73+12}{2} \times 62}{62} = \boxed{42.5}$$

$$n=2k \rightarrow 0, 1, 2, \dots, 9 \rightarrow a_0=1, a_1=2, a_2=4, a_3=8, \dots$$

$$n=2k+1 \rightarrow 1, 2, 3, 4, 5, \dots \rightarrow a_1=2, a_2=4, a_3=8, a_4=16, \dots$$

$$n=2k+2 \rightarrow 2, 3, 4, 5, 6, \dots \rightarrow a_2=1+a_1, a_3=1+a_2, a_4=1+a_3, \dots$$

$$\text{مجموع} = 1 + a + (1+1+2+2+2+2+2+2+2+2) = -20 + 18 = -2$$

$$t_n = an^2 + bn + c \rightarrow a = \frac{1}{2} \times 14 = 7 \rightarrow t_n = 7n^2 + bn + c \quad (3)$$

$$\begin{cases} t_0 = -\frac{1}{2} \times 14 + 0b + c = -7 + 0b + c = 19 \rightarrow 0b + c = 26 \\ t_1 = -\frac{1}{2} \times 14 + 1b + c = -7 + 1b + c = 17 \rightarrow 1b + c = 24 \end{cases} \rightarrow \begin{cases} 0b + c = 26 \\ 1b + c = 24 \end{cases} \rightarrow \begin{cases} c = 26 \\ b = -2 \end{cases} \rightarrow t_n = 7n^2 - 2n + 26$$

$$\rightarrow t_0 = -\frac{1}{2} \times 14 + 0b + c \rightarrow c = -1 \rightarrow t_n = -\frac{1}{2}n^2 + 8n - 1$$

$$\frac{t_{10}}{t_1} = \frac{-\frac{1}{2} \times 100 + 80 - 1}{-\frac{1}{2} \times 1 + 8 - 1} = \boxed{4}$$

$$a \rightarrow \text{مجموع} \rightarrow \text{مجموع} \rightarrow a_1 = t_1, a_2 = t_2 \rightarrow a + d = t + d', a + 2d = t + 2d' \rightarrow$$

$$\rightarrow a + 2d - a - d = t + 2d' - t - d' \rightarrow \frac{d}{1} = \frac{t}{1} \rightarrow t_0 = 0 \rightarrow t + 2d' = 0 \rightarrow t = -2d'$$

$$\frac{t_{10}}{t_1} = \frac{1 + 10d'}{1 + d'} = \frac{-2d' + 10d'}{-2d' + d'} = \frac{8d'}{-d'} = -8 \times \frac{d'}{d'} = \boxed{-8}$$

(4)

$$ta^r = aq_a + r q_a a \rightarrow q(a+d)^r = qa(a+rd) + ra(a+rd)$$

$$\rightarrow qa^r + rad + rd^r = aa^r + bld + ra^r + rad \rightarrow ra^r + ad - rd^r = 0$$

$$\rightarrow a^r + ad - rd^r = (a - rd)(r + d) = 0 \rightarrow a = \frac{rd}{r} \rightarrow \frac{a}{d} = \frac{r+1}{r}$$

$$\rightarrow a = \frac{rd}{r} = d \rightarrow \frac{a}{d} = 1$$

$$\rightarrow \frac{\frac{r}{r}d + rd}{d} = \frac{q}{r} = \boxed{\frac{r+1}{r}} \quad ! \rightarrow \frac{-rd + rd}{d} = \frac{d}{d} = \boxed{1}$$

(v)

$$(d \neq 0) \quad \text{جواب} \rightarrow a, b, c \rightarrow a, a+d, a+rd \quad \text{جواب} \rightarrow b, \frac{a}{r}, \frac{c}{r}$$

$$\rightarrow a+d, \frac{a}{r}, \frac{a+rd}{r} \rightarrow \frac{a^r}{r} = \frac{(a+d)(a+rd)}{r} \rightarrow a^r = a^r + rad + ad + rd^r \rightarrow rad + rd^r = 0$$

$$\rightarrow d(rad + rd^r) = 0 \rightarrow d \neq 0 \rightarrow ra + rd = 0 \rightarrow ra = -rd \rightarrow a = -\frac{r}{d}d$$

$$\rightarrow -\frac{r}{d}d + d, \frac{rd}{r}, \frac{-\frac{r}{d}d + rd}{r} \rightarrow \frac{d}{r}, \frac{-d}{r}, \frac{d}{r} \rightarrow r = -1 \rightarrow \boxed{r = -1}$$

(r ≠ 1)

$$\text{جواب} \rightarrow a, b, c \rightarrow \frac{b}{r}, b, br \quad , \quad \text{جواب} \rightarrow rb + ra, c$$

$$ra = rb + c \rightarrow \frac{rb}{r} = rb + br \rightarrow rb = rb + br^2 \rightarrow 0 = rb + br^2 \rightarrow r^2 + r - 1 = 0$$

$$\rightarrow (r+1)(r-1) = 0 \rightarrow r = 1 \times (\text{ممنوع}) \quad ! \quad r = -1 \rightarrow \frac{b}{-1}, b, -1b$$

$$\frac{a^r}{a^r} = \frac{ar^1}{ar^0} = r^r = (-1)^r = \boxed{-1}$$

(9)

$$\frac{ar^0}{(ar)^r} + \frac{ar}{ar} = \frac{ar^0}{ar^r} + \frac{r}{a} = \frac{r^r}{ar} + \frac{r}{a} = r \rightarrow \left(\frac{r}{a}\right)^r + \frac{r}{a} - r = 0$$

$$\rightarrow \left(\frac{r}{a} + r\right)\left(\frac{r}{a} - 1\right) = 0 \rightarrow \frac{r}{a} = 1 \rightarrow \frac{a}{r} = \boxed{1} \quad ! \quad \frac{r}{a} = -r \rightarrow \frac{a}{r} = \boxed{-\frac{1}{r}}$$

$$\left(\frac{a^r}{ar} = \frac{a}{r}\right)$$

(10)

$$a_r = \sqrt{a_r} \rightarrow ar^r = \sqrt{ar^r} = \sqrt{ar} \rightarrow ar = \sqrt{ar} \rightarrow a^r r^r = ar \rightarrow ar = 1$$

$$a_a = r r \quad ar = 1 = a_r \quad \frac{ar^r}{ar} = r^r = r r \rightarrow r = r \quad ar = r a = 1 \rightarrow a = \frac{1}{r}$$

$$\frac{1}{r} - a = \frac{1}{r} - \frac{1}{r} = \boxed{\frac{1}{r}}$$