۲۵

نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ۱۷ کلاس .....

۱

الف) $27^\circ = (x)\,rad \rightsquigarrow \frac{27}{180} = \frac{x}{\pi} \Rightarrow 27\pi = 180x \Rightarrow x = \frac{27\pi}{180} = \frac{3\pi}{20}$

ب) $125^\circ = (x)\,rad \rightsquigarrow \frac{125}{180} = \frac{x}{\pi} \Rightarrow 180x = 125\pi \Rightarrow x = \frac{125\pi}{180} = \frac{5\pi}{36}$

ج) $\frac{3\pi}{10}\,rad = (x)^\circ \rightsquigarrow \frac{\frac{3\pi}{10}}{\frac{\pi}{180}} = \frac{x}{180} \rightarrow 10x = 180 \times 3 \rightsquigarrow x = \frac{180\times 3}{10} = 54^\circ$

د) $\frac{4\pi}{9}\,rad = (x)^\circ \rightsquigarrow \frac{\frac{4\pi}{9}}{\frac{\pi}{180}} = \frac{x}{180} \rightsquigarrow 9x = 180\times 4 \rightsquigarrow x = \frac{180\times 4}{9} = 80^\circ$

۲

$10a^\circ$

$\frac{125a}{9}\,G \rightsquigarrow \frac{\frac{125a}{9}}{200} \rightsquigarrow \frac{125a}{9\times 200} = \frac{x}{180} \rightsquigarrow x = \frac{125a\times 180}{200\times 9} = 12/5a^\circ$

$\frac{a\pi}{8}\,rad \Rightarrow \frac{\frac{a\pi}{8}}{\pi} = \frac{a}{8} = \frac{y}{180} \rightarrow y = \frac{180a}{8} = 22/5a^\circ$

$10a + 12/5a + 22/5a = 45a = 180 \rightsquigarrow a = \frac{180}{45} \Rightarrow a = 4^\circ$

۳

الف) $\cos 40^\circ \cos 30^\circ - \sin 40^\circ \sin 30^\circ - \tan 45^\circ + 2\cot 45^\circ =$

$\left(\frac{1}{2}\times\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{3}}{2}\times\frac{1}{2}\right) - 1 + 2(1) = 1$

$\tan = \frac{\sin}{\cos}$, $\cot = \frac{\cos}{\sin}$

$\tan 45^\circ = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1 \;/\; \cot 45^\circ = 1$

ب) $\frac{\tan^2 30^\circ + \tan^2 45^\circ + \tan^2 60^\circ}{\cot 30^\circ - \cot 60^\circ} = \frac{\frac{3}{9} + 1 + 3}{\sqrt{3} - \frac{\sqrt{3}}{3}} = \frac{\frac{3+9+27}{9}}{\frac{3\sqrt{3}-\sqrt{3}}{3}} = \frac{\frac{39}{9}}{\frac{2\sqrt{3}}{3}} = \frac{\frac{13}{3}}{\frac{2\sqrt{3}}{3}} = \frac{13\sqrt{3}}{6}$

$\frac{13}{3}\times\frac{\sqrt{3}}{2\sqrt{3}}$

$\tan^2 30^\circ = \left(\frac{\sqrt{3}}{3}\right)^2 = \frac{1}{3}$, $\tan^2 60^\circ = (\sqrt{3})^2 = 3$

$\tan 45^\circ = 1$, $\cot 30^\circ = \sqrt{3}$, $\cot 60^\circ = \frac{\sqrt{3}}{3}$

۴

$-\sin 30^\circ \cos 60^\circ + \cos 30^\circ \sin 60^\circ = \sin^2\theta$

$\left(-\frac{1}{2}\times\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2}\right) = \frac{-1}{4} + \frac{3}{4} = \frac{2}{4} = \frac{1}{2} \rightsquigarrow \sin^2\theta = \frac{1}{2} \rightsquigarrow \sin\theta = \pm\frac{1}{\sqrt{2}} \Rightarrow \pm\frac{\sqrt{2}}{2}$

$\cos^2 = 1 - \sin^2 = 1 - \frac{1}{2} = \frac{1}{2} \rightarrow \cos\theta = \pm\frac{\sqrt{2}}{2}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = 1$

۵

$\frac{2\tan 30^\circ(1-\tan^2 30^\circ)}{(1-\cot^2 60^\circ)^2} = \tan\theta \rightsquigarrow \frac{2\left(\frac{\sqrt{3}}{3}\right)\left(1-\frac{1}{9}\right)}{\left(1-\frac{3}{9}\right)^2} = \frac{\frac{2\sqrt{3}}{3}}{1-\frac{3}{9}} = \frac{\frac{2\sqrt{3}}{3}}{\frac{6}{9}} = \frac{2\sqrt{3}}{3}\times\frac{3}{2} = \sqrt{3}$

$\star\ 90^\circ > \theta$

$\tan\theta = \sqrt{3} \rightsquigarrow \frac{\sin\theta}{\cos\theta} = \sqrt{3} \rightsquigarrow \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3} \Rightarrow \theta = 60^\circ \rightarrow rad(x) = 60^\circ$

$\frac{60}{180} = \frac{x}{\pi} \Rightarrow 180x = 60\pi$

$x = \frac{60\pi}{180} = \frac{\pi}{3}$

$\tan 30^\circ = \frac{\sqrt{3}}{3} \rightarrow \left(\frac{\sqrt{3}}{3}\right)^2 = \frac{3}{9}$ $\quad \cot^2 60^\circ = \left(\frac{\sqrt{3}}{3}\right)^2 = \frac{3}{9}$

۶

$\tan\theta = ۵$ ⟶ sin , cos ?

$1+\tan^2\theta = \frac{1}{\cos^2\theta}$ ⟶ $1+۲۵ = ۲۶ = \frac{1}{\cos^2\theta} \Rightarrow \cos^2\theta = \frac{1}{۲۶} \to \cos\theta = \pm\frac{1}{\sqrt{۲۶}}$

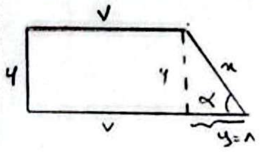
$\sin^2 = 1-\cos^2 = \frac{۲۶-1}{۲۶} = \frac{۲۵}{۲۶}$ ⟶ $\sin\theta = \pm\frac{۵}{\sqrt{۲۶}}$

$$\frac{۳\sin\theta-\cos\theta}{\sin\theta-۴\cos\theta} = \frac{۳(\pm\frac{۵}{\sqrt{۲۶}})-(\pm\frac{1}{\sqrt{۲۶}})}{\pm\frac{۵}{\sqrt{۲۶}}-۴(\pm\frac{1}{\sqrt{۲۶}})} = \frac{\frac{\pm۱۵-(\pm1)}{\sqrt{۲۶}}}{\frac{\pm۵-(\pm۴)}{\sqrt{۲۶}}} = \begin{cases}\frac{۱۵-1}{۵-۴} = ۱۴\\ \frac{-۱۵+1}{-۵+۴} = \frac{-۱۴}{-1} = ۱۴\end{cases}$$

جواب آخر (۱۴)

۲

۷



$\sin\alpha = \frac{۶}{۱۰}$

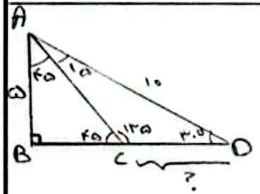
⟶ $\sin = \frac{\text{مقابل}}{\text{وتر}} = \frac{۶}{x} = \frac{۶}{۱۰}$ ⟶ $x = ۱۰$

$y^2 = ۱۰^2 - ۶^2 = ۱۰۰ - ۳۶ = ۶۴ \Rightarrow y = ۸$

$P = ۶ + ۷ + ۱۰ + ۸ + ۷ = ۳۸$

۲

۸



AB ⟶ ۳۰° روبرو ⟶ $\frac{AB}{۱۰} = \frac{1}{۲}$, $۲AB = ۱۰ \to AB = ۵$

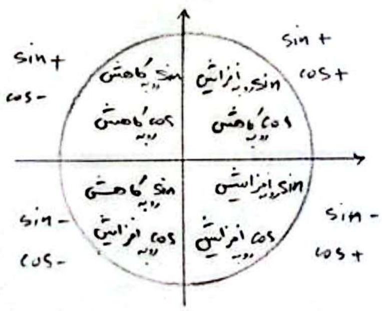
ABC ⟶ متساوی‌الساقین ⟹ $AB = BC = ۵$

BD (فیثاغورس): $BD^2 = ۱۰^2 - ۵^2 = ۱۰۰ - ۲۵ = ۷۵ \to BD = \sqrt{۷۵} = ۵\sqrt{۳}$

$CD = BD - BC = ۵\sqrt{۳} - ۵$
فاکتورگیری ⟶ $۵(\sqrt{۳}-1)$

۲

۹



الف) ناحیه سوم

ب) ناحیه دوم

۳

۱۰

$\tan x = \frac{1}{۳}$ ⟶ $\tan^2 x = \frac{1}{۹}$

$1+\tan^2 x = \frac{1}{\cos^2 x}$ ⟶ $1+\frac{1}{۹} = \frac{۱۰}{۹} = \frac{1}{\cos^2 x}$ ⟶ $\cos^2 x = \frac{۹}{۱۰} \Rightarrow \cos x = \pm\frac{۳}{\sqrt{۱۰}}$ (ناحیه سوم ⟵)

$\sin^2 x = 1-\cos^2 x = 1-\frac{۹}{۱۰} = \frac{1}{۱۰}$ ⟶ $\sin x = \pm\frac{1}{\sqrt{۱۰}}$ (ناحیه سوم)

$\sin x = -\frac{1}{\sqrt{۱۰}}$

۳