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نام و نام خانوادگی ......فاطمه احمدی...... پاسخنامه تشریحی تکلیف شماره ..۵.. کلاس ....دهم تجربی A....

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$a_1+a_2+a_3=21 \rightsquigarrow a_1+a_3+4=21 \rightsquigarrow (a_1+a_3=17)$ ①

$a_1\times a_2\times a_3=64 \rightsquigarrow \frac{a_2}{q}\times a_2\times a_2q=64 \rightsquigarrow a_2^3=64 \rightsquigarrow a_2=4 \rightsquigarrow a_1\times a_3\times 4=64 \rightsquigarrow (a_1\times a_3=16)$ ②

① $a_1+a_3=a_1+a_1q^2=17 \rightsquigarrow a_1+\frac{16}{a_1}=\frac{a_1^2+16}{a_1}=17 \rightsquigarrow a_1^2+16=17a_1 \rightsquigarrow a_1^2-17a_1+16=0$

$(a_1-16)(a_1-1)=0 \rightsquigarrow a_1=1\ /\ 16$

② $a_1\times a_3=a_1^2q^2=16 \rightsquigarrow a_1q^2=\frac{16}{a_1}$

if $a_1=1 \rightsquigarrow 1,4,16 \rightsquigarrow q=\frac{16}{4}=4$ (صعودی) : غیرقابل قبول

if $a_1=16 \rightsquigarrow 16,4,1 \rightsquigarrow q=\frac{1}{4}$

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$x^2+4,\ 2x,\ x^2-2 \to$ هندسی نزولی

$* \ b^2=ac \rightsquigarrow 4x^2=(x^2+4)(x^2-2) \Rightarrow 4x^2=x^4+2x^2-8 \rightsquigarrow x^4-2x^2-8=0 \to (x^2-4)(x^2+2)=0$

$x^2-4=0$

$x^2=4 \to x=\pm 2$

if $x=2 \rightsquigarrow 8,4,2 \to q=\frac{2}{4}=\frac{1}{2}$ ✱

if $x=-2 \rightsquigarrow 8,-4,2 \to q=\frac{-1}{2} \rightsquigarrow (q>0)$ شرط نزولی : غیرقابل قبول

$S_7=a_1\left(\frac{1-q^n}{1-q}\right)=8\left(\frac{1-(\frac{1}{2})^7}{1-\frac{1}{2}}\right)=8\left(\frac{\frac{127}{128}}{\frac{1}{2}}\right)=8\times\frac{127}{64}=\frac{127}{8}$

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$a_1=243$

$1+q+q^2+q^3+q^4=\frac{121}{81}$

$S_5=a_1+a_1q+a_1q^2+a_1q^3+a_1q^4=a_1(1+q+q^2+q^3+q^4)=243\left(\frac{121}{81}\right)=363$

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$1,\underbrace{\dots}_{5},64 \rightsquigarrow$ حسابی : $a_7=a_1+6d=64 \rightsquigarrow 6d=63 \rightsquigarrow d=\frac{63}{6}=10/5 \rightsquigarrow A=a_4=a_1+3d=1+3(10/5)=32/5$

هندسی : $a_7=a_1q^6=64 \rightsquigarrow q^6=64 \rightsquigarrow q=\pm 2 \rightsquigarrow B=a_4=a_1q^3=1\times(\pm 2)^3=\pm 8$

$A+B \to \begin{cases} \text{if } B=+8 \Rightarrow 32/5+8=40/5 \\ \text{if } B=-8 \Rightarrow 32/5-8=24/5 \end{cases}$

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$-24,\ \frac{-95}{4},\dots$ (حسابی) $\rightsquigarrow d=-\frac{95}{4}-(-24)=\frac{-95}{4}+24=\frac{1}{4} \rightsquigarrow$ ۱۰۱ جمله : $a_1+100d=-24+100(\frac{1}{4})=1$

$128, a_2, \dots$ (هندسی)

$a_8=1 \rightsquigarrow a_1q^7=1 \to 128\times q^7=1 \to q^7=\frac{1}{128} \rightsquigarrow q=\frac{1}{2}$

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$a_3, a_7, a_9 \to (a_1+2d), (a_1+6d), (a_1+8d)$

$(a_1+6d)^2 = (a_1+2d)(a_1+8d) \rightsquigarrow \cancel{a_1^2}+36d^2+12a_1d = \cancel{a_1^2}+8a_1d+2a_1d+16d^2$

$20d^2+2a_1d=0 \rightsquigarrow 2d(10d+a_1)=0 \begin{cases} 2d=0 \to d=0 \\ 10d+a=0 \to d=\frac{-a}{10} \end{cases}$ ۶

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$a_2, a_4, a_8 \to \underset{2a_1}{(a_1+d)}, \underset{4a_1}{(a_1+3d)}, \underset{8a_1}{(a_1+7d)}$

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$a_1 = \frac{1}{4}$

$(a_1+3d)^2 = (a_1+d)(a_1+7d) = \cancel{a_1^2}+9d^2+6a_1d = \cancel{a_1^2}+a_17d+a_1d+7d^2$

$2d^2-2da_1=0 \to 2d(d-a_1)=0 \begin{cases} d=0 & \text{غ ق ق} \\ d=a_1 \Rightarrow^{*} d=\frac{1}{4} \end{cases}$ ۷

$* \ \frac14, \frac12, 1, 2, \dots \to q=\frac{2}{1}=\boxed{2}$

$a_{10}=a_1q^9=\frac14\times2^9=\frac{1}{2^2}\times2^9=2^7=\boxed{128}$

$3a_2, 2a_3, a_4 \to$ حسابی : $3a_1q, 2a_1q^2, a_1q^3 \qquad 2a_1q^2=\frac{3a_1q+a_1q^3}{2}$

$4a_1q^2=3a_1q+a_1q^3 \to a_1q^3-4a_1q^2+3a_1q=0 \rightsquigarrow \underline{a_1q}(q^2-4q+3)=0$

$q=0$ غ ق ق $\qquad \hookrightarrow (q-1)(q-3)=0$

$q=1$ یا 3 ۸

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$a_n=2, \frac74, \dots \rightsquigarrow d=\frac{7-8}{4}=\frac{-1}{4} \Rightarrow a_4=2+3(-\frac14)=\frac54 \ / \ a_8=a_1+7d=2+7(-\frac14)=\frac14 \ /$

$a_{13}=a_1+12d=2+12(-\frac14)=-1$

هندسی

$\frac54+x, \frac14+x, -1+x \rightsquigarrow (\frac14+x)^2=(\frac54+x)(x-1)= \frac1{16}+\cancel{x^2}+\frac12x = \cancel{x^2}+\frac54x-x-\frac54-\frac{20}{16}$

$\frac14x+\frac{21}{16}=0 \to \frac{x}{4}=\frac{-21}{16} \to x=\frac{-84}{16}=\frac{-21}{4}$ ۹

$t_n=\frac{-16}{4}, -\frac{20}{4}, \frac{-25}{4} \rightsquigarrow -4, -5, -\frac{25}{4} \rightsquigarrow q=\frac{-25/4}{-20/4}=\frac{25}{20}=\frac54 \Rightarrow q=\frac54$

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a_1, a_4, a_7 (هندسی) $\to a_1, a_1q^3, a_1q^6 \qquad a_4-a_1=\frac{a_7-a_4}{8}$

$a_1+a_4+a_7=73$

$a_1(1+q^3+q^6)=73 \qquad \cancel{a_1}(q^3-1)=\frac{a_1q^3(q^3-1)}{8}, \ \frac{q^3}{8}=1 \rightsquigarrow q^3=8 \to q=2$

$a_1(\underbrace{1+8+64}_{73})=73 \to 73a_1=73$

$a_1=1$ ۱۰

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$1, 8, \dots \to d=8-1=\boxed{7}$