الناز دادخواه دهم جمع ها A 11 ۱۸:۳۰ تا ۱۶:۲۰

-۱

$a + aq + aq^2 = 21 \longrightarrow a(1+q+q^2) = 21$

$a \times aq \times aq^2 = 64 \rightarrow a^3q^3 \rightarrow (a\times q)^3 = 64 \rightarrow a\times q = \sqrt[3]{64} = \boxed{+4} \quad a = \frac{4}{q}$

$\frac{4}{q}(1+q+q^2) = 21 \longrightarrow \frac{4}{q} + 4 + 4q = 21 \rightarrow 4 + 4q + 4q^2 = 21q \rightarrow 4 - 17q + 4q^2 = 0$

$\underbrace{+4}_{a} \underbrace{-17q}_{b} \underbrace{+4q^2}_{c} = 0 \quad \Delta = b^2 - 4ac \rightarrow \frac{-b \pm \sqrt{\Delta}}{2a} \rightarrow \frac{-(-17) \pm \sqrt{(-17)^2 - (4\times 4\times 4)}}{2\times 4} = \frac{+17 \pm 15}{8} = \nearrow \frac{2}{8} = \frac{1}{4}$ ✓ چون قسمت غیرحسابی

$289 - 64 = \sqrt{225} = 15$

$\searrow \cancel{\frac{32}{8}} = 4$ غ ق

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$b^2 = a\times c \longrightarrow (2x)^2 = (x^2+4)(x^2-2) \longrightarrow 4x^2 = x^4 - 2x^2 + 4x^2 - 8 \rightarrow 4x^2 = x^4 + 2x^2 - 8$

$x^4 - 2x^2 - 8 = 0 \xrightarrow{y = x^2} y^2 - 2y - 8 = 0 \longrightarrow (y-4)\times(y+2) = 0$

$y = 4 \longrightarrow x^2 = 4 \rightarrow +2 = x, \ -2 = x$

$y = -2 \rightarrow x^2 = -2 \rightarrow$ جواب ندارد X

درصورتی یک دنباله با قدر نسبت مثبت نزولی می شود که قدر نسبت عددی بین ۱ و ۰ باشد.

$a_1 = x^2 + 4 \rightarrow 8$ ✓ نزولی است

$a_2 \rightarrow 4 \rightarrow 2x$

$a_3 \rightarrow 2 \rightarrow x^2 - 2$

حالت ۱) $x = 2$ $\longrightarrow 8, 4, 2 \rightarrow q = \frac{1}{2}$ ($\times\frac{1}{2}$, $\times\frac{1}{2}$)

حالت دوم = نزولی نیست X: $a_1 = 8, \ a_2 = -4, \ a_3 = 2 \longrightarrow 8, -4, 2 \rightarrow q = -\frac{1}{2}$

نتیجه: $a_1 = 8$, $q = \frac{1}{2}$

S_n جمله هندسی $S_7 = \frac{a_1(q^n - 1)}{-1+q} = \frac{8\left(\frac{1}{2}^7 - 1\right)}{\frac{1}{2}} = \frac{8\left(\frac{1}{128} - \frac{128}{128}\right)}{-\frac{1}{2}} = \frac{8\times\left(-\frac{127}{128}\right)}{-\frac{1}{2}} = \boxed{\frac{127}{8}} = S_7$

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$a_1 = 243 \qquad 1+q+q^2+q^3+q^4 = \frac{121}{81} \longrightarrow a + aq + aq^2 + aq^3 + aq^4 = ?$

$a(1+q+q^2+q^3+q^4) \longrightarrow \cancel{243}^{3}\left(\frac{121}{\cancel{81}_1}\right) = 363$

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1 $a_1, a_2, a_3, \boxed{a_4}^{A}, a_5, a_6, a_7 \longrightarrow a_7 = 64, \ a_1 = 1 \rightarrow d = \frac{64-1}{6} = \frac{63}{6} = 10/5$

$a_4 = a_1 + 3d \rightarrow 1 + (3\times 10/5) = 1 + 31/5 = 32/5 = A$

2 $a_1, a_2, a_3, \boxed{a_4}^{aq^3}, a_5, a_6, a_7 = \rightarrow a_7 = 64 \rightarrow aq^6 = 64 \rightarrow q = \sqrt[6]{64} \nearrow 2, \searrow -2$

$B \rightarrow 1\times 2^3 = 8 \longrightarrow 32/5 + 8 = 40/5$

$\searrow 1\times(-2)^3 = -8 \longrightarrow 32/5 + (-8) = 24/5$

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$\frac{-95}{4} - (-24) = \frac{-95}{4} + \frac{24}{1} = \frac{-95+96}{4} = +\frac{1}{4} \to a_1 + 100d \to -24 + 100\times(\frac{1}{4}) = +1$

$a_1 q^7 = +1 \to 128 q^7 = 1 \to r^7 = q^7 = \frac{1}{128} \to r = \frac{1}{2}$

۶-

$a_3 = a+2d \quad a_7 = a+6d \quad a_9 = a+8d \xRightarrow{\text{در هندسی}} (a+6d)^2 = (a+2d)(a+8d)$

$a^2 + 12ad + 36d^2 = a^2 + 10ad + 16d^2 \to 2ad + 20d^2 = 0 \to 2d(a+10d) = 0$

دو حالت داریم: ① $a \neq 0 ,\ d = 0$ ← در این حالت همه جملات برابر میشوند

② $a + 10d = 0 \to 10d = -a \to d = \frac{-a_1}{10}$

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$a_1 = \frac{1}{4} a_1$

$k_1 = a_2 = a_1 d$

$k_2 = a_4 = a_1 + 3d$

$k_3 = a_8 = a_1 + 7d$

$\to \frac{a_4}{a_2} = \frac{a_8}{a_4} = r \to \frac{a_1+3d}{a_1+d} = \frac{a_1+7d}{a_1+3d}$

$(a_1+3d)(a_1+3d) = (a_1+7d)\times(a_1+d) \to a_1^2 + 6a_1d + 9d^2 = a_1^2 + 7a_1d + a_1d + 7d^2$

$a_1^2 + 6a_1d + 9d^2 = a_1^2 + 8a_1d + 7d^2 \to 2d^2 - 2a_1d = 0 \to 2d(d - a_1) = 0$

دوباره دو حالت داریم اما سوال گفته متمایز پس فقط شرط ۲ مورد قبول است.

$d - a_1 = 0 \to d = a_1$

$k_1 = 2a_1 \quad k_2 = 4a_1 \to \frac{4a_1}{2a_1} = 2 = r \xrightarrow{n=10} a_1 r^9 \to \frac{1}{4} \times 2^9 \Rightarrow k_{10} = \times 2^{9-2}$

$\leftarrow k_{10} = 128$

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$B_1 = 3k_2 = 3k_1 r \ ,\ B_2 = 2k_3 = 2k_1 r^2 \ ,\ B_3 = k_4 = k_1 r^3 \to A_2 - A_1 = A_3 - A_2$ شرط در دنباله حسابی

$2k_1r^2 - 3k_1r = k_1r^3 - 2k_1r^2$

$\cancel{k_1r}(2r-3) = \cancel{k_1r}(r^2 - 2r) \to 2r - 3 = r^2 - 2r \to r^2 - 4r + 3 = 0 \to (r-3)(r-1) = 0$ ق ع

چون گفته غیر ثابت ۳=r مورد قبول است.

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$\frac{7}{4} - 2 = \frac{7}{4} - \frac{8}{4} = -\frac{1}{4} = d \ ,\ a_4 = 2 - \frac{3}{4} = \frac{5}{4} \ ,\ a_8 = 2 - \frac{7}{4} = \frac{1}{4} \ ,\ a_{12} = 2 - 3 = -1$

$b_1 = a_4 + k = \frac{5}{4} + k \ ,\ b_2 = a_8 + k = \frac{1}{4} + k \ ,\ b_3 = a_{12} + k = -1 + k$

$(\frac{1}{4}+k)^2 = (\frac{5}{4}+k)\times(-1+k) \to \frac{1}{16} + \frac{1}{2}k + k^2 = -\frac{5}{4} + \frac{1}{4}k + k^2 \to \frac{1}{4}k = -\frac{21}{16} \Rightarrow k = \frac{-21}{4}$

$b_1 = \frac{5}{4} + (\frac{-21}{4}) = -4$

$b_2 = \frac{1}{4} + (-\frac{21}{4}) = -5$

$\to \frac{b_2}{b_1} = \frac{-5}{-4} = \boxed{\frac{5}{4} = r}$

-۱۰

a و a_4 و a_7 ، $a_1 + a_4 + a_7 = 73$

$a_1 = t_1 \quad a_4 = t_2$ و $a_7 = t_{20}$

$$a_4 - a_1 = \frac{a_7 - a_4}{8} \rightarrow a_1(q^3 - 1)$$

$$\frac{a_1 q^3 (q^3 - 1)}{8} = 1 = \frac{q^3}{8} \rightarrow q^3 = 8 \rightarrow q = 2$$

$$a_1 + a_1 q^3 + a_1 q^6 = a_1(1 + q^3 + q^6) = a_1(8 + 1 + 64) = 73 a_1 = 73$$

$$a_1 = 1$$

$a_1 = t_1 = 1 \quad a_4 = t_2 = 8 \quad t_2 - t_1 = 8 - 1 = 7$