به نام خدا

تکلیف ۱۵

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۱)

$\frac{a}{q} \times a \times aq = 64 \to a^3 = 64 \to a = 4$

$\frac{a}{q} + a + aq = 21 \to \frac{4}{q} + 4 + 4q = 21 \xrightarrow{\times q} 4q^2 + 4q + 4 = 21q$

$4q^2 - 17q + 4 = 0$

$\Delta = 289 - 64 = 225$

$x = \frac{17 \pm \sqrt{225}}{8} \to 4$ ✗ $\to \boxed{\frac{1}{4} = q}$

۲)

$b^2 = ac \to 4x^2 = x^4 + 2x^2 - 8$

$0 = x^4 - 2x^2 - 8$

$x^2 = t \to t^2 - 2t - 8 = 0 \to (t-4)(t+2) = 0 \to 4, -2$

$x^2 = 4 \to x = \pm 2$, $x^2 \neq -2$

$+2 \to a_1 = 8 \quad a_2 = 4 \quad a_3 = 2$ ✓ $\to q = \frac{1}{2}$

$-2 \to 8 \quad -4 \quad 2$ ✗

$S_7 = a\left(\frac{1-q^7}{1-q}\right) \to 8 \times \left(\frac{1-\frac{1}{128}}{1-\frac{1}{2}}\right)$

$8 \times \frac{\frac{127}{128}}{\frac{1}{2}} = \frac{127 \times 2}{16} = 15/875$

۳)

$1 + q + q^2 + q^3 + q^4 = \frac{121}{81} \xrightarrow{\times a_1}$

$a_1(1 + q + q^2 + q^3 + q^4) = \frac{121}{81} a_1$

$a_1 + aq + aq^2 + aq^3 + aq^4 = \frac{121}{81} \times 243$

$a_1 + aq + aq^2 + aq^3 + aq^4 = 363$

۴)

$d = \frac{b-a}{n+1} \to \frac{64-1}{6} = 10/5$

$b = \frac{a+c}{2} \to \frac{64+1}{2} \to \frac{65}{2} = 32/5 \to A$

$\frac{b}{a} = q^{n+1} \to \frac{64}{1} = q^6 \to q = \pm 2 \to b^2 = ac \to 64 \times 1 = b^2 \to b = \pm 8$

$A + B = 32/5 \pm 8 = 40/5,\ 24/5$

۵)

$a_1 + (n-1)d \to -24 + (n-1) + \frac{1}{4} \to -24 + 100\left(+\frac{1}{4}\right) \to 1$

$d = -\frac{95}{4} - (-24) = +\frac{1}{4}$

$a_1 \times q^{n-1} \to 128 \times q^7 = 1 \to q^7 = \frac{1}{128} \to q = \frac{1}{2}$

۶)

$a_1 + 2d$

$a_1 + 6d$

$a_1 + 8d$

$(a_1 + 8d)(a_1 + 2d) = (a_1 + 6d)^2$

$\cancel{a^2} + 10ad + 16d^2 = \cancel{a^2} + 12ad + 36d^2$

$-2a\cancel{d} = 20d^2$

$-2a = 20d$

$a = -10d \to d = -\frac{1}{10}a$

۷)

$a_1 + d \to 2d$

$a_1 + 3d \to 4d \quad (a_1 + 3d)^2 = (a_1 + d)(a_1 + 7d)$

$a_1 + 7d \to 8d \quad \cancel{a^2} + 9d^2 + 6ad = \cancel{a^2} + 8ad + 7d^2$

$\cancel{2}d^{\cancel{2}} = \cancel{2}a\cancel{d} \to d = a$

$q = 2$

$a_1 = \frac{1}{4} \quad a_{10} = a_1 q^9 \to \frac{1}{4} \times 2^9 \to \frac{1}{\cancel{2^2}} \times 2^{\cancel{9}\,7} = 128$

۸)

$3a_2 \to 3a_1q$

$2a_3 \to 2a_1q^2$

$a_4 \to a_1q^3$

$\frac{3a_1q + a_1q^3}{2} = 2a_1q^2 \to 3a_1q + a_1q^3 = 4a_1q^2$

$\cancel{a_1}\cancel{q}(3 + q^2) = 4\cancel{a_1}q^{\cancel{2}}$

$3 + q^2 = 4q$

$q^2 - 4q + 3 = 0 \to 1,\ 3$

۹)

$a+3d+x$ (1) $\qquad a+7d+x$ (2) $\qquad a+12d+x$ (3) $\qquad d=-\frac{1}{4} \qquad a=2$

(1): $2+3(-\frac{1}{4})+x \to \frac{5}{4}+x \to \frac{5}{4}-\frac{21}{4}=-4$

(2): $\frac{1}{4}+x \to \frac{1}{4}-\frac{21}{4}=-5$

(3): $-1+x$

$b^2=ac \to (\frac{1}{4}+x)^2=(x-1)(x+\frac{5}{4})$

$\cancel{x^2}+\frac{1}{16}+\frac{1}{2}x=\cancel{x^2}+\frac{1}{4}x-\frac{5}{4}$

$-\frac{21}{16}=\frac{1}{4}x \to \boxed{x=-\frac{21}{4}}$

$q^{m-n}=\frac{a_m}{a_n}=q^{2-1}=\frac{-5}{-4}=q=\frac{5}{4}$ جواب آخر

۱۰)

(1) $\qquad$ (2) $\qquad$ (10)

$a_1+a_1q^3+a_1q^9=73 \xrightarrow{q=2} a_1+8a_1+64a_1=73a_1=73 \to a_1=1$

$d=\frac{a_2-a_1}{1}=a_1q^3-a_1=a_1(q^3-1) \longrightarrow d=a_1(q^3-1) \to 1(8-1)=\boxed{7}$ جواب آخر

$9d=a_1q^9-a_1=a_1(q^9-1) \to d=\frac{a_1(q^9-1)}{9} \longrightarrow \frac{\cancel{a_1}(q^9-1)}{9}=\cancel{a_1}(q^3-1)$

$\frac{(q^3+1)\cancel{(q^3-1)}}{9}=\cancel{q^3-1}$

$q^3+1=9$

$q^3=8 \to q=2$