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نام و نام خانوادگی: رها موسوی مدی پاسخنامه شماره ۱۴ تکلیف کلاس دختر A

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$a_1+a_2+a_3=21$, $a_1\times a_2\times a_3=64 \rightarrow \frac{a_2}{q}\times a_2\times a_2q=a_2^3=64 \rightarrow a_2=4$

$a_1\times a_3=16 \rightarrow a_1^2q^2=16$

$a_1+a_3=17 \rightarrow a_1+aq^2=17 \xrightarrow{a_1q^2=16} a_1+\frac{16}{a_1}=17 \rightarrow a_1^2-17a_1+16=0 \rightarrow a_1=1 \text{ یا } 16$

$a_1=16$, $a_2=4 \rightarrow q=\frac{1}{4}$ ✓

$a_1=1$, $a_2=4 \rightarrow q=4$ غ ق ق $(q\neq 1)$

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x^2+4 , $2x$, $x^2-2 \rightarrow 4x^2=(x^2+4)(x^2-2)=x^4+2x^2-8 \rightarrow x^4-2x^2-8=0 \rightarrow (x^2-4)(x^2+2)=0$

$x^2-4=0 \rightarrow x=\pm 2$ ، $x^2+2>0$

$x=2 \rightarrow 8, 4, 2 \rightarrow q=\frac{1}{2}$

$x=(-2) \rightarrow 8, -4, 2 \rightarrow q=\frac{-1}{2}$ غ ق ق $(q>0)$

$S_\infty=8\left(\frac{1-(\frac{1}{2})^\infty}{1-\frac{1}{2}}\right)=\frac{8}{\frac{1}{2}}=\frac{128}{64}\times 8=\frac{128}{8}$

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$1+q+q^2+q^3+q^4=\frac{121}{16}$, $a_1=256$

$S_5=a_1+a_1q+a_1q^2+a_1q^3+a_1q^4=a_1(1+q+q^2+q^3+q^4)=256\times\frac{121}{16}=3872$

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$a_1=1$, $a_7=64 \rightarrow a_7=a_1+6d=64 \rightarrow 1+6d=64 \rightarrow d=\frac{63}{6}=\frac{21}{2}=10.5 \rightarrow a_4=1+3(10.5)=32.5$

$t_1=1$, $t_7=64 \rightarrow t_7=t_1q^6=64 \rightarrow 1\times q^6=64 \rightarrow q=\pm 2 \rightarrow t_4=\pm 8$

$\begin{cases} A=32.5 \\ B=\pm 8 \end{cases} \Rightarrow \begin{cases} A+B=32.5+8=40.5 \\ A+B=32.5-8=24.5 \end{cases}$

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$-24, \frac{-95}{4}, \dots \rightarrow d=\frac{-95}{4}-(-24)=\frac{1}{4} \rightarrow t_{101}=t_1+100d=-24+\frac{100}{4}=1$

$128, a_2, \dots \rightarrow a_8=a_1q^7=128q^7=1 \rightarrow q=\frac{1}{2}$

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$a_3, a_7, a_9 \longrightarrow a_1+2d, a_1+6d, a_1+8d$

$\hookrightarrow (a_1+6d)^2=(a_1+2d)(a_1+8d) \longrightarrow a_1^2+12a_1d+36d^2=a_1^2+10a_1d+16d^2 \rightarrow 20d^2+2a_1d=0$

$10d^2+a_1d=0 \rightarrow d(10d+a_1)=0 \rightarrow d=0$

$10d+a_1=0 \rightarrow d=\frac{-a_1}{10}$

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$a_2, a_4, a_8 \longrightarrow \underbrace{a_1+d}_{2a_1}, \underbrace{a_1+3d}_{4a_1}, \underbrace{a+7d}_{8a_1} \longrightarrow q=2 \rightarrow a_{10}=a_1q^9=\frac{1}{4}\times 2^9=128$

$(a_1+3d)^2=(a_1+d)(a_1+7d) \longrightarrow a_1^2+9a_1d+9d^2=a_1^2+7a_1d+8a_1d \rightarrow 2d^2-2da_1=0$

$d=a_1$

$d=0$ غ ق ق

(٣)

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$3a_2, 2a_3, a_4 \longrightarrow 3a_1q, 2a_1q^2, a_1q^3$

$\hookrightarrow 2a_1q^2-3a_1q=a_1q^3-2a_1q^2 \rightarrow a_1q(2q-3)=a_1q(q^2-2q) \rightarrow 2q-3=q^2-2q \rightarrow q^2-4q+3=0$

$(q-1)(q-3)=0 \Rightarrow q=1$ ق ق ق

$q=1$ غ ق ق $\Rightarrow \boxed{q=3}$

(٣)

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$2, \frac{7}{4}, \dots \longrightarrow d=\frac{-1}{4} \longrightarrow a_4=2+\frac{3(-1)}{4}=\frac{5}{4}, a_8=2+\frac{7(-1)}{4}=\frac{1}{4}, a_{13}=2+\frac{12(-1)}{4}=(-1)$

$\hookrightarrow a_4+K, a_8+K, a_{13}+K \longrightarrow \frac{5}{4}+K, \frac{1}{4}+K, (-1)+K \rightarrow (\frac{1}{4}+K)^2=(\frac{5}{4}+K)(K-1)$

$\frac{1}{16}+K^2+\frac{K}{2}=K^2-\frac{5}{4}+\frac{K}{4} \rightarrow \frac{K}{4}=\frac{-21}{16}$

$\Longrightarrow K=\frac{-21}{4}$ $\{-4, -a, -\frac{25}{4} \Rightarrow q=\frac{5}{4}$

(٣)

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$a_1, a_4, a_7 \quad , a_1+a_4+a_7=39$

$a_1=t_1, a_4=t_2, a_7=t_{10} \qquad a_4-a_1=\frac{a_7-a_4}{8} \rightarrow a_1(q^3-1)=\frac{a_1q^3(q^3-1)}{8} \rightarrow 1=\frac{q^3}{8} \rightarrow q=2$

(١٠٣)

$a_1+a_1q^3+a_1q^6=a_1(1+q^3+q^6)=a_1(1+8+64)=73a_1=39 \rightarrow a_1=1$

$a_1=t_1=1 \qquad t_2=a_4=2^3\times 1=8 \longrightarrow t_2-t_1=8-1=7$

(١) $q=1$ $3a=39 \rightarrow a=\frac{39}{3}$ $aq^3-a=d \rightarrow \frac{39}{3}-\frac{39}{3}=0 \rightarrow d=0$