نام و نام خانوادگی ....[illegible]....   پاسخنامه تشریحی تکلیف شماره ..۱۵.. کلاس ..A..........

۱

$a + ar + ar^2 = 21$

$a \cdot ar \cdot ar^2 = 64 \rightarrow ar = 4$

$\frac{4}{r} + 4 + 4r = 21$

$\frac{4}{r}(1 + r + r^2) = 21$

$4 + 4r + 4r^2 = 21r \rightarrow 4r^2 - 17r + 4 = 0 \qquad r = \frac{17 \pm \sqrt{225}}{8} = \begin{cases} 4 = r \\ \frac{1}{4} = r \end{cases} \rightarrow \boxed{r = \frac{1}{4}}$

$17 \times 17 = 119 + 170 = 289$

۲

$4x^2 = x^4 + 2x^2 - 8$

$x^4 - 2x^2 - 8 = 0 \rightarrow (x-2)(x+2)(x^2+2) \rightarrow x = \begin{cases} 2 \\ -2 \end{cases}$

$2, 4, 8, \dots \rightarrow 2\left(\frac{2^7 - 1}{2 - 1}\right) = 2 \times \overset{127}{\cancel{(128 - 1)}} = \boxed{254}$

$2, -4, 8, \dots \rightarrow 2\left(\frac{(-2)^7 - 1}{-2 - 1}\right) = 2 \times \frac{\overset{-129}{\cancel{-128 - 1}}}{-3} = 2 \times 43 = \boxed{86}$

۳

$a + aq + aq^2 + aq^3 + aq^4 = S_5$

$S_5 = a(1 + q + q^2 + q^3 + q^4)$

$S_5 = \overset{3}{\cancel{243}} \times \frac{121}{\cancel{81}} = \boxed{363}$

۴

$\frac{64 + 1}{2} = \frac{65}{2} = 32.5 = A$

$q^2 = \frac{64}{1} \rightarrow q = \pm 8 = B$

$A + B \begin{cases} 32.5 + 8 = \boxed{40.5} \\ 32.5 - 8 = \boxed{24.5} \end{cases}$

۵

$-\frac{45}{4} + 24 = \frac{1}{4} = d \rightarrow a_{101} = \underset{-24}{a_1} + \overset{25}{100}\left(\frac{1}{4}\right) \Rightarrow a_{101} = 1$

$b_8 = a_1 r^{n-1} \rightarrow 1 = 128 \times r^7 \rightarrow r^7 = \frac{1}{128} \rightarrow \boxed{r = \frac{1}{2}}$

۶

$a+2d \,,\, a+6d \,,\, a+8d \rightarrow b^2 = ac$

$a^2+36d^2+12ad = a^2+16d^2+10ad$

$20d^2 = -2ad \quad (d \neq 0)$

$20d = -2a$

$10d = -a$

$\boxed{d = \frac{-a}{10}}$

۷

$a+d \,,\, a+3d \,,\, a+7d$

$a^2+9d^2+6ad = a^2+7d^2+8ad$

$2d^2-2ad=0$

$2d(d-a)=0 \quad d\neq 0 \rightarrow a=d$

$2a \,,\, 4a \,,\, 8a \qquad a_{10} = \frac{1}{4} \times 2^9 = 2^7 = 128$

$\frac{4a}{2a} = 2 = r$

۸

$3ar \,,\, 2ar^2 \,,\, ar^3$

$4\cancel{a}r^2 = \cancel{a}(3r+r^3)$

$4r^2 = 3r+r^3$

$4r = 3+r^2$

$r^2-4r+3=0 \rightarrow r = \frac{4\pm\sqrt{4}}{2} = 3 \,,\, \cancel{1} \rightarrow r=3$

۹

$\frac{7}{4}-2 = -\frac{1}{4} = d \rightarrow a_n = 2+(n-1)(-\frac{1}{4})$

$a_4 = 2+(-\frac{3}{4}) = \frac{7}{4} \qquad x =$ قدرنسبت

$a_8 = 2+(-\frac{7}{4}) = \frac{1}{4}$

$a_{12} = 2+(-3) = -1$

$\frac{7}{4} + \underbrace{\frac{29}{4}}_{x} = \frac{36}{4} = 9$

$\frac{1}{4} + \frac{29}{4} = \frac{30}{4} = 7.5$

$\frac{7.5}{9} = \boxed{\frac{2.5}{3} = q}$

$(\frac{1}{4}+x)^2 = (\frac{7}{4}+x)(-1+x)$

$\frac{1}{16}+x^2+\frac{x}{2} = x^2+\frac{3x}{4}-\frac{7}{4} \rightarrow \frac{1}{16}+\frac{28}{16} = \frac{3x}{4}-\frac{2x}{4} \rightarrow \frac{29}{16} = \frac{x}{4} \rightarrow x = 7.25$

۱۰

$a + ar^3 + ar^6 = 73 \qquad a(1+r^3+r^6) = 73 \rightarrow 73a = 73 \rightarrow a=1$

$a + a+d + a+9d = 73 \qquad 3a+10d = 73 \rightarrow 3+10d = 73 \rightarrow 10d = 70 \rightarrow d=7$

$d = ar^3-a$

$ar^6 = a+9a(r^3-1)$

$r^6 = 1+9r^3-9$

$r^6 = 9r^3-8 \rightarrow r^6-9r^3+8=0 \qquad r^3 = \frac{9\pm\sqrt{49}}{2} \rightarrow r^3 = \begin{cases} 8 \rightarrow r=2 \\ 1 \rightarrow \cancel{r=1} \end{cases}$