

$$\star^2 a(1+q+q^2) = 21 \rightarrow \frac{4}{q}(1+q+q^2) = 21 \rightarrow 4(1+q+q^2) = 21q \rightarrow 4 + 4q + 4q^2 = 21q$$

$$\star^1 a \cdot aq \cdot aq^2 = 44 \rightarrow a^3 q^3 = 44 \rightarrow aq = 4 \rightarrow a = \frac{4}{q}$$

$$4q^2 - 17q + 4 = 0 \rightarrow \begin{cases} q = 4 \rightarrow \text{حسابی} \\ q = \frac{1}{4} \rightarrow \text{غیرحسابی} \end{cases}$$

$$\frac{a_1}{a_2} = \frac{a_3}{a_4} \rightarrow \frac{2x}{x^2+4} = \frac{x^2-2}{2x} \rightarrow 4x^2 = x^4 + 2x^2 - 8 \rightarrow x^4 - 2x^2 - 8 = 0 \rightarrow x = 2$$

$$a_1 = 8, a_2 = 4, a_3 = 2 \rightarrow q = \frac{4}{8} = \frac{1}{2}$$

$$S_v = 8 \left(\frac{1 - \frac{1}{2^v}}{1 - \frac{1}{2}} \right) = 8 \times \frac{127}{128} \times 2 = \frac{127}{8} \rightarrow \text{جواب}$$

$$S_\infty = a(1+q+q^2+q^3+q^4) = 243 \times \frac{121}{81} = 363$$

$$d = \frac{94-1}{5+1} = \frac{93}{6} = 15.5$$

$$a_n = 1, 11/5, 22, 32/5, 43, 53/5, 64$$

$$A = 32/5, B = 8$$

$$A+B = 32/5 + 8 = 48/5$$

$$q^4 = 94 \rightarrow q = 2$$

$$a_n = 1, 2, 4, 8, 16, 32, 64$$

$$128 \times q^v = 1 \rightarrow q^v = \frac{1}{128} \rightarrow q = \frac{1}{2}$$

$$\star^1 d = \frac{99}{4} - \frac{95}{4} = \frac{4}{4} = 1$$

$$a_{101} = -24 + 1/100 \times \frac{1}{2} = 1$$

$$(a+4d)^2 = (a+2d)(a+6d) \rightarrow a^2 + 12ad + 36d^2 = a^2 + 10ad + 12d^2$$

$$2ad + 24d^2 = 0 \rightarrow 2d(a+12d) = 0 \rightarrow \begin{cases} d=0 \\ d = -\frac{10}{a} \end{cases}$$

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$$a_2 = \frac{1}{4}q, a_4 = \frac{1}{4}q^3, a_8 = \frac{1}{4}q^7$$

$$2 \times \frac{1}{4}q^3 = \frac{1}{4}q + \frac{1}{4}q^7 \rightarrow 2q^3 = q + q^7 \rightarrow q^7 - 2q^3 + q = 0 \rightarrow q(q^6 - 2q^2 + 1) = 0 \rightarrow q^6 - 2q^2 + 1 = 0 \xrightarrow{[t=q^2]} t^3 - 2t + 1 = 0$$

$$t^3 - 2t + 1 = 0 \rightarrow \text{if } t=1 \rightarrow 1-2+1=0 \rightarrow q^2=1 \rightarrow q=\pm 1 \rightarrow q=-1$$

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$$a_n = a_1 q^{n-1} \rightarrow a_{10} = \frac{1}{4} \times (-1)^9 = -\frac{1}{4}$$

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$$aq, 2aq^2, aq^3 \rightarrow 2(2aq^2) = aq + aq^3 \rightarrow 4q^3 = q + q^3$$

$$q^3 - 4q^3 + q = 0 \rightarrow q(q^2 - 4q + 1) = 0 \rightarrow q \neq 0$$

$$q = \frac{4 \pm \sqrt{(-4)^2 - 4 \times 1 \times 1}}{2} = \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3} = q$$

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$$a_n = 2, \frac{1}{4}, \dots \rightarrow d = \frac{1}{4} - 2 = -\frac{7}{4}$$

$$a_4 = 2 + 3(-\frac{7}{4}) = 2 - \frac{21}{4} = -\frac{17}{4}, a_8 = 2 + 7(-\frac{7}{4}) = 2 - \frac{49}{4} = -\frac{45}{4}$$

$$(\frac{1}{4} + k)^2 = (-\frac{17}{4} + k)(-\frac{45}{4} + k) \rightarrow k^2 + \frac{1}{4}k + \frac{1}{16} = k^2 + \frac{1}{4}k - \frac{21}{4} \rightarrow \frac{1}{4}k = -\frac{21}{4} \Rightarrow k = -\frac{21}{4}$$

$$q = \frac{a_8 + k}{a_4 + k} \Rightarrow \frac{\frac{1}{4} - \frac{21}{4}}{-\frac{17}{4} - \frac{21}{4}} = \frac{-\frac{20}{4}}{-\frac{38}{4}} = \frac{10}{19} = q$$

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$$a + aq^2 + aq^4 = \sqrt{3} = a(1 + q^2 + q^4) \mid q^4 = 9q^2 - 8 \rightarrow q^4 - 9q^2 + 8 = 0 \xrightarrow{[q^2=t]} t^2 - 9t + 8 = 0$$

$$d = aq^2 - a = a(q^2 - 1)$$

$$t^2 - 9t + 8 = 0 \rightarrow (t-1)(t-8) = 0 \rightarrow \begin{cases} t=1 \rightarrow q=1 \\ t=8 \rightarrow q^2=8 \rightarrow q=\sqrt{8} \end{cases}$$

$$\text{if } q=1 \rightarrow a(1+q^2+q^4) = \sqrt{3} \rightarrow a(1+1+1) = \sqrt{3} \rightarrow a = \frac{\sqrt{3}}{3}, d = a(q^2-1) = \frac{\sqrt{3}}{3}(1-1) = 0$$

$$\text{if } q=\sqrt{2} \rightarrow 1+q^2+q^4 = 1+2+4=7 \rightarrow \sqrt{3} = 7a \rightarrow a = \frac{\sqrt{3}}{7}, d = a(q^2-1) = \frac{\sqrt{3}}{7}(2-1) = \frac{\sqrt{3}}{7}$$

$$\boxed{d=0}$$

$$\boxed{d=\frac{\sqrt{3}}{7}}$$

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