۱۹

کوثر عبدلی

① $a_1 \times a_2 \times a_3 = 64 \Rightarrow \frac{a_2}{q} \times a_2 \times a_2 q = 64 \Rightarrow a_2^3 = 64 \Rightarrow a_2 = 4$

$\frac{4}{q} + 4 + 4q = 21 \Rightarrow 4 + 4q^2 = 17q \Rightarrow 4q^2 - 17q + 4 = 0$ ۱٫۵

$\Delta = (-17)^2 - 4(4)(4) = 225 \Rightarrow q = \frac{+17 \pm 15}{8} \begin{cases} 4 \\ \frac{1}{4} \end{cases}$

غیرقابل قبول چون نزولی است

② $a_1 = x^2 + 4 \quad a_2 = 2x \quad a_3 = x^2 - 2$ ۲

$(2x)^2 = (x^2+4)(x^2-2) \Rightarrow 4x^2 = x^4 + 2x^2 - 8$

$\Rightarrow x^4 - 2x^2 - 8 = 0 \qquad x^2 = y$

$y^2 - 2y - 8 = 0 \rightarrow y = \frac{+2 \pm \sqrt{36}}{2} = \begin{cases} 4 \\ -2 \end{cases}$

$q > 0 \Rightarrow 4 \rightsquigarrow x^2 = 4 \Rightarrow x = \pm 2$

$x = 2 \Rightarrow 8, 4, 2$ ✓ نزولی ✓ $\quad q > 0$

$\times\frac{1}{2} \quad \times\frac{1}{2}$

$\rightsquigarrow S_7 = a_1 \frac{1 - (\frac{1}{2})^7}{1 - (\frac{1}{2})} = 8 \times \frac{\frac{127}{128}}{\frac{1}{2}} = \frac{\cancel{8} \times 127}{\cancel{64}_{8}}$

$= \boxed{\frac{127}{8}}$

③ $S_5 = \frac{1(1-q^5)}{1-q} = 1 + q + q^2 + q^3 + q^4 = \frac{121}{81} = \frac{11^2}{9^2}$ ۲

$\cancel{1+q+q^2+q^3+q^4 - \frac{121}{81} = 0 \Rightarrow 81 + 81q + 81q^2 + 81q^3 + 81q^4 - 121 = 0}$

$\cancel{\Rightarrow 81q^4 + 81q^3 + 81q^2 + 81q - 40 = 0}$

$\frac{1-q^5}{1-q} = \frac{121}{81} \rightsquigarrow 1 - q^5 = \frac{121}{81}(1-q) \Rightarrow 1 - q^5 = \frac{121}{81} - \frac{121}{81}q$

$\Rightarrow q^5 - \frac{121}{81}q + \frac{121}{81} - 1 = 0 \Rightarrow q^5 - \frac{121}{81}q + \frac{40}{81} = 0 \rightarrow \frac{1}{3}$

$\Rightarrow 81(\frac{1}{3})^5 - 121(\frac{1}{3}) + 40$

$81 = 3^4 , \; 121 = 11^2 \qquad \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}$

$\Rightarrow 81(\frac{1}{243}) - \frac{121}{3} + 40 \Rightarrow \frac{1 - 121}{3} + 40$

$1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} = \frac{121}{81}$

$\Rightarrow \frac{-120}{3} + 40 \Rightarrow -40 + 40 = 0$ ✓

$\Rightarrow \boxed{\frac{1}{3}}$

$\Rightarrow \cancel{S_5 = 243(1 - \frac{1}{243})}$

ادامه صفحه بعد

کنکور سراسری

$n=۵ , q=\frac{1}{۳} , a=۲۴۳$

$\Rightarrow S_5 = \frac{۲۴۳(1-\frac{1}{۲۴۳})}{\quad -1} \Rightarrow \frac{۲۴۳(\frac{۲۴۳-1}{۲۴۳})}{\frac{۳-1}{۳}} \Rightarrow \frac{۲۴۳(\frac{۲۴۲}{۲۴۳})}{\frac{۲}{۳}}$

$\Rightarrow S_5 = \frac{۲۴۲}{\frac{۲}{۳}} \Rightarrow ۲۴۲ \times \frac{۳}{۲} \Rightarrow \frac{۲۴۲\times ۳}{۲} \Rightarrow ۱۲۱\times ۳$

$S_5 = ۳۶۳$

س ۴

$a_1 , a_2 , a_3 , \overset{A}{a_4} , a_5 , a_6 , a_7$

$a_n = a_1+(n-1)d \quad , \quad a_1 = 1 , a_7 = ۶۴ , n = ۷$

$\Rightarrow ۶۴ = 1+۶d \Rightarrow \boxed{d = ۱۰/۵}$

$\Rightarrow a_4 = a_1+(n-1)d \Rightarrow a_4 = 1+۳\times ۱۰/۵ = \boxed{۳۲/۵}$

$a_n = a_1 . r^{n-1} \quad a_1 = 1 \quad a_7 = ۶۴$

$۶۴ = 1\times r^6 \rightarrow r = \pm ۲$

$r+ \rightarrow a_1 = 1 , r = ۲ \Rightarrow B = a_4 = ۲^3 = \boxed{۸}$

$r = -۲ \Rightarrow B = a_4 = (-۲)^3 = \boxed{-۸}$

$\Rightarrow A = ۳۲/۵ , B = \pm ۸.$

$۳۲/۵ + ۸ = ۴۰/۵$

$۳۲/۵ - ۸ = ۲۴/۵$

س ۵

$a_1 = -۲۴ = -\frac{۹۶}{۴} , a_2 = -\frac{۹۵}{۴} \quad d = a_2 - a_1 \Rightarrow -\frac{۹۵}{۴} - (-\frac{۹۶}{۴})$

$d = -\frac{۹۵}{۴} + \frac{۹۶}{۴} = \boxed{\frac{1}{۴}}$

$a_{101} = -۲۴ + ۲۵ = \boxed{1}$

$a_1 = ۱۲۸ , a_8 = 1$

$a_8 = a_1 \times r^7 \Rightarrow r^7 = \frac{1}{۱۲۸} = \boxed{\frac{1}{۲}}$

PAPCO

کوثر عادلی

۲ ۴

$a_3 = \cancel{a_1 + (3-1)d} = a_1 + 2d$

$a_7 = \cancel{a_1 + (7-1)d} = a_1 + 6d$

$a_9 = \cancel{a_1 + (9-1)d} = a_1 + 8d$

$(a_7)^2 = (a_3)(a_9) \Rightarrow (a_1+6d)^2 = (a_1+2d)(a_1+8d)$

$\Rightarrow a_1^2 + 12a_1 d + 36d^2 = a_1^2 + 8a_1 d + 2a_1 d + 16d^2$

$\Rightarrow a_1^2 + 12a_1 d + 36d^2 = a_1^2 + 10a_1 d + 16d^2$

$\Rightarrow 12a_1 d + 36d^2 = 10a_1 d + 16d^2$

$\Rightarrow 12a_1 d - 10a_1 d + 36d^2 - 16d^2 = 0 \Rightarrow 2a_1 d + 20d^2 = 0$

$\Rightarrow \boxed{2d(a_1+10d) = 0} \Rightarrow 2d = 0 \Rightarrow d = 0$ (۱)

$a_3 = a_7 = a_9 = a_1$ تمام جملات برابر هم هستند

$r = \frac{a_1}{a_1} = 1$ به شرطی که $a_1 \neq 0$; اگر $a_1 = 0$ [illegible] r [illegible]

$d = -\frac{a_1}{10} \Leftarrow 10d = -a_1 \Leftarrow a_1 + 10d = 0$ (۲)

$r = \frac{a_7}{a_3}, \ d = -\frac{a_1}{10} \Rightarrow r = \frac{a_1 + 6(-\frac{a_1}{10})}{6a_1} = \frac{4a_1}{8a_1}$

$a_1 \neq 0 \Rightarrow r = \frac{4}{8} = \frac{1}{2}$

$r = 1$ اگر $d = 0 , a_1 \neq 0$

$a_1 \neq 0 , d = -\frac{a_1}{10} \Rightarrow r = \frac{1}{2}$

اگر $a_1 = 0$ [illegible]

PAPCO

$a_2 = a_1 + d$, $a_4 = a_1 + 3d$, $a_8 = a_1 + 7d$

$(a_4)^2 = (a_2)(a_8)$. $d \neq 0$

$(a_1 + 3d)^2 = (a_1 + d)(a_1 + 7d)$

$\Rightarrow a_1^2 + 2(a_1)(3d) + 9d^2 = a_1^2 + (a_1)(7d) + (d)(a_1) + (d)(7d)$

$\Rightarrow 2d^2 - 2a_1 d = 0 \Rightarrow 2d(d - a_1) = 0$

$2d = 0 \Rightarrow d = 0$

$d - a_1 = 0 \Rightarrow d = a_1$

$d = 0 \Rightarrow a_2 = a_4 = a_8 = a_1$ ✗ متمایز

$\Rightarrow \boxed{d = a_1}$

$q = \frac{a_4}{a_2}$, $q = \frac{a_8}{a_4}$

$a_2 = a_1 + d = a_1 + a_1 = 2a_1$

$a_4 = a_1 + 3d = a_1 + 3a_1 = 4a_1$

$a_8 = a_1 + 7d = a_1 + 7a_1 = 8a_1$

$\Rightarrow q = \frac{4a_1}{2a_1} \longrightarrow \boxed{q = 2} \xrightarrow{a_1 \neq 0}$

$b_1 = \frac{1}{4}$, $q = 2$

$\Rightarrow b_n = b_1 \cdot q^{n-1}$

$n = 10\ (b_{10}) \rightarrow b_{10} = \frac{1}{4} \times 2^9 = 512 \Rightarrow \boxed{b_{10} = 128}$

کوشش عالی (۳)

۸) ۵

$t_1=a,\ t_2=aq,\ t_3=ar^2,\ t_4=ar^3$

$\cancel{3t_2=3ar,\ 2t_3=2ar^2,\ t_4=ar^3}$

$3t_2,\ 2t_3,\ t_4=(3ar,\ 2ar^2,\ ar^3)$

$2(2ar^2)=3ar+ar^3 \Rightarrow 4r=3+r^2$

$\Rightarrow r^2-4r+3=0 \qquad (r-1)(r-3)=0 \quad r=1,\ r=3$

$r\neq 1 \longrightarrow$ غ ق

$r=3$ ✓

۹) ۳

$d=\dfrac{\frac{7}{4}-2}{2-1}=\dfrac{-1}{4} \qquad a_1=2$

$a_4=a_1+3d=2+\dfrac{-3}{4}=\dfrac{5}{4}$

$a_8=a_1+7d=2+\dfrac{-7}{4}=\dfrac{1}{4}$

$a_{13}=a_1+12d=2+(-3)=-1$

$b_1=\dfrac{5}{4}+k,\ b_2=\dfrac{1}{4}+k,\ b_3=-1+k$

$\Rightarrow \left(\dfrac{1}{4}+k\right)^2=\left(\dfrac{1}{4}\right)^2+2\left(\dfrac{1}{4}\right)k+k^2$

$\dfrac{1}{16}+\dfrac{1}{2}k+k^2=-\dfrac{5}{4}+\dfrac{1}{4}k+k^2$

$\Rightarrow \dfrac{2}{4}k-\dfrac{1}{4}k=\dfrac{-20}{16}-\dfrac{1}{16} \Rightarrow \dfrac{1}{4}k=-\dfrac{21}{16}$

$\Rightarrow k=\dfrac{-21}{4}$

$b_1=-4,\ b_2=-5,\ b_3=\dfrac{-25}{4}$

$q=\dfrac{-5}{-4}=\dfrac{5}{4}$ و $\dfrac{\frac{-25}{4}}{-5}=\dfrac{5}{4}$

a_1 , a_4 , a_7

a , aq^3 , aq^6 $\qquad$ $a^2 q^6 = a^2 q^6$ $\qquad$ $d = q^3$

$a(1+q^3+q^6) = 73$ $\qquad$ $2aq^3 = a(1+q^6)$

$\frac{1+q^6}{2}$ $\qquad$ $q^3 = \frac{1+q^6}{2}$

$a(1+q$

$\frac{aq^3(1-q^3)}{8} = q^3$

$aq^3(1-q^3) = 8q^3$

$a_1 , a_4 , a_7 \Rightarrow a , aq^3 , aq^6$ $\qquad$ 15،

$A = a$ $\qquad$ $A \quad A+d \quad A+9d$ $\qquad$ ۲

$a + aq^3 + aq^6 = 72$

مشابه $\Rightarrow$ $\boxed{\begin{array}{l} aq^3 = a + d \\ d = aq^3 - a \end{array}}$, $\boxed{\begin{array}{l} aq^6 = a + 9d \\ 9d = aq^6 - a \\ \Rightarrow d = \frac{aq^6 - a}{9} \end{array}}$

$aq^3 - a = \frac{aq^6 - a}{9} \Rightarrow 9aq^3 - 9a = aq^6 - a$

$\Rightarrow a(q^6 - 9q^3 + 8) = 0$

$q^3 = x \Rightarrow x^2 - 9x + 8 = 0$

$\Rightarrow (x-1)(x-8) = 0 \Rightarrow x = 8 , x = 1$

پس $q^3 = 1 \Rightarrow \overset{\text{غیرممکن}}{q = 1}$, $q^3 = 8 \Rightarrow q = 2$

$\hookrightarrow A + A + A = 3A = 72 \Rightarrow A = \frac{72}{3} \Rightarrow d = 0$

$\rightarrow aq^6 = aq^3 = a \rightarrow q = 1$

$\Rightarrow a = A + 9d = A + d a = A \Rightarrow 9d = 0 \Rightarrow d = 0$ ✗ غیرمجاز

$q = 2 \Rightarrow a + a + a = 72 \Rightarrow a = \frac{72}{3}$

PAPCO

$\Rightarrow 72$ $\cancel{+\frac{72}{2} + d + \frac{72}{2} + 9d = 72}$

کوئیز عالی

~~$q=1 \to a + a + a = 3a = 73 \to a = \frac{73}{3}$~~

~~$A_1 + d = a + d = A$~~

امتیاز (۱۰)

$\boxed{q=2} \to q^3 = 8 , q^6 = 64$

$a + aq^3 + aq^6 = 73$

$a + 8a + 64a = 73$

$73a = 73 \Rightarrow \boxed{a = 1}$

$a_1 = 1 , a_4 = 8 , a_7 = 64$

$A_2 = A_1 + d \to 8 = 1 + d \Rightarrow d = 7$

$A_{10} = A_1 + 9d \Rightarrow 64 = 1 + 63 \Rightarrow 64 = 64$ ✓

$q = 1 \to d = 0$

$\boxed{q = 2 \longrightarrow d = 7}$ صحیح است.