۱۸,۷۵

کلاس : دختران دهم A

(آنیا شرجی)

۱- (۵)

$a_1 \times a_2 \times a_3 = 64$ $\frac{a}{q} \times a \times aq = 64 = a^3 \to a = 4$

$a_1 + a_2 + a_3 = 21$ $(\frac{4}{q} + 4 + 4q = 21) \times q$

$\to 4 + 4q + 4q^2 = 21q \to 4q^2 - 17q + 4 = 0$ $q = \frac{17 \pm \sqrt{289 - 64}}{2 \times 4} = \frac{17+15}{8} = 4 \ , \ \frac{17-15}{8} = \frac{1}{4}$

$q = \frac{1}{4}$

۲- (۵)

$x^2+4, 2x, x^2-2$ $(2x)^2 = (x^2+4)(x^2-2)$

$8, 4, 2 \longrightarrow q = \frac{1}{2}$ $4x^2 = x^4 - 2x^2 + 4x^2 - 8$

$(x^2)^2 - 2x^2 - 8 = 0$ $x^2 = \frac{2 \pm \sqrt{4-(-32)}}{2} =$

$x^2 = \frac{2+6}{2} = 4 \to x = 2$ ، $\frac{2-6}{2} = -2$ ✗

$S_7 = a\left(\frac{q^n - 1}{q-1}\right) = 8\left(\frac{(\frac{1}{2})^7 - 1}{\frac{1}{2} - 1}\right) = 8 \times \frac{1 - \frac{1}{128}}{\frac{1}{2}} = \frac{8 \times \frac{127}{128}}{\frac{1}{2}} =$

$\to \frac{2 \times 127}{16} = 15{,}875$

۳- (۵)

$a_1 = 243$

$S_5 = a(1 + q + q^2 + q^3 + q^4) = \longrightarrow \frac{243 \times 121}{81} = 363$

$a = 243$ ، $\frac{121}{81}$

۵- (۵)

حسابی $-24, \frac{-95}{4}, \dots$ حسابی $a_{101} = a_8$ هندسی $q = ?$

هندسی $128, a_2, \dots$

$a + (n-1)d = -24 + 100 \times \frac{1}{4} = 1$

$a_8 = 128 \times q^7 = 1$ $q^7 = \frac{1}{128} \to q = \frac{1}{2}$

۴- (۵)

واسطه حسابی $= \frac{64-1}{5+1} = \frac{63}{6} = 10{,}5$ $a_4 = a + (n-1)d = 1 + (4-1) \times 10{,}5 = 32{,}5$

واسطه هندسی : $q^{n+1} = \frac{64}{1} \to q = \pm 2$ $a_4 = aq^3 = 1 \times 2^3 = 8$ یا -8

$A + B = 40{,}5$ ✓

$A + B = 32{,}5 - 8 = 24{,}5$

۶)

$$\begin{cases} a_3 = a + 2d = a \\ a_7 = a + 6d = aq \\ a_9 = a + 8d = aq^2 \end{cases}$$

$(a+6d)^2 = (a+8d)(a+2d)$ ۱,۲۵

$a^2 + 36d^2 + 12da = a^2 + 10da + 16d^2$

$20d^2 = -2da \rightarrow 10d = -a \rightarrow \boxed{a = -10d}$

$\boxed{d = \frac{a}{-10}}$

$d = 0$

قابل قبوله چون نقطه‌ها نمیزند

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۷)

$a_2 = a + d = a$

$a_4 = a + 3d = aq$

$a_8 = a + 7d = aq^2$

$(a+3d)^2 = (a+7d)(a+d) = 10a$

$a^2 + 9d^2 + 6da = a^2 + 8da + 7d^2$

$2d^2 = 2da \rightarrow a = d$

(۵)

$\frac{1}{4}, \frac{1}{2}, 1, 2, 4, 8, 16, 32, 64, (128)$ جمله دهم

$q = \frac{4a}{2a} = 2$

$aq^9 = \frac{1}{4} \times 512 = \boxed{128}$

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۸)

(۵)

$2aq, aq^2, aq^3$

$2aq^2 = \frac{3aq + aq^3}{2}$

$4aq^2 = 3aq + aq^3 \rightarrow q^3 - 4q^2 + 3q = 0$

$4q^2 = 3q + q^3$

$q(q^2 - 4q + 3) = 0$

$q = \frac{4 \pm \sqrt{16-12}}{2} = \frac{4+2}{2} = 3, \quad \frac{4-2}{2} = 1$

$\rightarrow \boxed{q = 3}$

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۹)

$a_4 = 2 + 3 \times \frac{-1}{4} = 1.25 + d$

$a_8 = 2 + 7 \times \frac{-1}{4} = \frac{1}{4} = 0.25 + d$ (۳)

$a_{13} = 2 + 12 \times \frac{-1}{4} = -1 + d$

$(0.25 + d)^2 = (-1 + d)(1.25 + d)$

$(0.25)^2 + d^2 + 0.5d = -1.25 - d + 1.25d + d^2$ ($0.25d$)

$0.0625 + 1.25 = 0.25d - 0.5d$

$1.3125 = -0.25d \rightarrow \boxed{d = -5.25}$

$-4, -5, -6.25, \ldots \rightarrow \boxed{q = 1.25}$

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۱۰)

$a_1 + a_4 + a_7 = a + aq^3 + aq^6 = 73$

$\downarrow a, \quad a + 3d, \quad a + 9d$

$aq^3 = a + d$

$aq^6 = a + 9d$

$\rightarrow d = a(q^3 - 1)$ (۱۱۵)

$aq^6 = a(1 + 9(q^3 - 1)) = a(9q^3 - 8)$

$q^6 = 9q^3 - 8 \quad \begin{cases} q = 1 \\ q = 8 \end{cases}$

$a(1 + 8 + 64) = 73$

$73a = 73 \rightarrow a = 1$

پاسخ نهایی: $8 - 1 = \boxed{7}$

$q = 1$

$73a = 73$

$a = \frac{73}{73}$

$d = aq^3 - a$

$\frac{73}{73} - \frac{73}{73} = 0$