۱- ص $\quad$ ۱۸/۲۵

$a, aq, aq^2 \qquad (\text{وسط})^3 = 64 \to \text{وسط} = aq = 4 \to a = \frac{4}{q}$

$a(1+q+q^2) = 21 \to \frac{4}{q}(1+q+q^2) = 21 \times q \to 4q^2 - 17q + 4 = 0$

$\Delta = b^2 - 4ac = (-17)^2 - 4\times4\times4 = 225$

$q = \frac{17 \pm 15}{8} \Rightarrow q = \frac{32}{8} = 4 \ ,\ q = \frac{2}{8} = \boxed{\frac{1}{4}}$

۲- ص

$(2x)^2 = (x^2-2)(x^2+4) \to 4x^2 = x^4 + 2x^2 - 8 \to x^4 - 2x^2 - 8 = 0 \qquad (x^2-4)(x^2+2) = 0 \to x = \pm 2$

$(x-2)(x+2)$

$x = 2 \qquad 8, 4, 2, \dots$

$x = -2 \qquad 8, -4, 2, \dots$

$S_7 = 8 \times \frac{1-(\frac{1}{2})^7}{1-\frac{1}{2}} = 16 - 16 \times \frac{1}{128} = \frac{127}{8}$

۳- ص

$1 + q + q^2 + q^3 + q^4 = \frac{121}{81} \xrightarrow{\times a} a + aq + aq^2 + aq^3 + aq^4 = \frac{121}{\cancel{81}} \times \cancel{243}^{\,3} = 363$

۴- ۱۱/۵

$d = \frac{64-1}{5+1} = 10.5 \to 1, 11.5, 22, \underset{A}{32.5}, 43, 53.5, 64$

$32.5 + 8 = 40.5$

$q^{6} = \frac{64}{1} = \to q = 2 \to 1, 2, 4, \underset{B}{8}, 16, 32, 64$

$B = -8$

$q = -2$

$A + B = 24.5$

۵- ص

$d = \frac{1}{4} \to a_n = -24 + \frac{1}{4}(n-1) = \frac{1}{4}n - 24.25 \to a_{101} = \frac{1}{4}\times 101 - 24.25 = 1$

$b_n = bq^{n-1} = \frac{128}{2^7} \times q^7 \to (2q)^7 = 1^7 \to 2q = 1 \to q = \frac{1}{2}$

۶- ۱۱/۲۵

$a+2d, \ a+6d, \ a+8d$

$(a+6d)^2 = (a+2d)(a+8d) \to a^2 + 12ad + 36d^2 = a^2 + 10ad + 16d^2 \to 20d^2 = -2ad$

$a_2 \quad a_2 + 2d \quad a_2 + 6d \qquad$ درسته چون نقطه متمایز $\leftarrow d = 0 \qquad d = -\frac{1}{10}a$

۷- ص

$(a_2+2d)^2 = a_2(a_2+6d) \to \cancel{a_2^2} + 4d^2 + 4a_2d = \cancel{a_2^2} + 6a_2d \xrightarrow{d\neq0} 4d^2 = 2a_2d \to d = \frac{a_2}{2}$

$r = \frac{a_4}{a_2} = \frac{a_2+2d}{a_2} = \frac{a_2 + 2(\frac{a_2}{2})}{a_2} = 2 \to b_{10} = b_1 \times r^9 = \frac{1}{4} \times 2^9 = 2^7 = 128$

۸- ص

$3aq, \ 2aq^2, \ aq^3 \to 2aq^2 = \frac{aq^3 + 3aq}{2} \to 4aq^2 = aq^3 + 3aq$

$4q = q^2 + 3 \to q^2 - 4q + 3 = 0$

$(q-1)(q-3)$

$q < \begin{matrix} 1 \times \text{ غیرقابل‌قبول} \\ 3 \checkmark \end{matrix}$

۹_

$+x \quad a_4, a_8, a_{13}$

$a+3d+x,\ a+7d+x,\ a+12d+x \xrightarrow{(a+x)=k} (k+7d)^2=(k+3d)(k+12d)$

$k^2+49d^2+14kd=k^2+15kd+36d^2 \to 13d^2=kd$

$13d=k$

$$r=\frac{\overset{13d}{k}+7d}{\underset{13d}{k}+3d}=\frac{20d}{16d}=\boxed{\frac{5}{4}}$$

۱۰_

$a_1, a_4, a_7 \longrightarrow a, ar^3, ar^6$

$\downarrow \quad \downarrow \quad \downarrow$

$a_1 \quad a_2 \quad a_{10} \longrightarrow a, a+d, a+9d \to (a+d)^2=a(a+9d) \Rightarrow \cancel{a^2}+d^2+2ad=\cancel{a^2}+9ad$

$d^2=7ad$

$d=7a$

$\frac{d}{7}=a$

$a+a+d+a+9d=3a+10d=73$

$\frac{3d}{7}+10d=\frac{73d}{7}=73$

$\boxed{d=7}$

۱/۵

$q=1$

$3a=73 \longrightarrow a=\frac{73}{3}$

$d=aq^3-a \longrightarrow \frac{73}{3}-\frac{73}{3}=0$