عارفه سعادت نیا

ده دقیقه A

(۱)

$a_1 + a_2 + a_3 = 21$ $a_1 \times a_2 \times a_3 = 64$

$\frac{4}{q} + 4 + 4q = 21$ $a_2^3 = 64$

$q = 4$ یا $\frac{1}{4}$ $a_2 = 4$

نزولی $\rightarrow q = \frac{1}{4}$

(۲)

$a_2^2 = a_1 \times a_3$

$(2x)^2 = (x^2+4)(x^2-2) \rightarrow 4x^2 = x^4 + 2x^2 - 8$

$x^4 - 2x^2 - 8 = 0$

$(x-4)(x+2) = 0$

$x^2 = -2$ ×

$x^2 = 4 \rightarrow x = \pm 2$

$x = -2$: $a_1 = 8$ $a_2 = -4$, $a_3 = 2 \rightarrow r = -\frac{1}{2}$ ×

$x = 2$: $a_1 = 8$, $a_2 = 4$, $a_3 = 2 \rightarrow r = \frac{1}{2}$ ✓

(۳)

$a_1 + (a_1 q) + (a_1 q^2) + (a_1 q^3) + (a_1 q^4) = 243 + 243q + 243q^2 + 243q^3 + 243q^4 =$

$243 (1 + q + q^2 + q^3 + q^4) = 363$

$\frac{121}{81}$

(۴)

$\frac{64+1}{2} = 32/5 = A$ $\sqrt{64 \times 1} = 8 = B$

$A + B = 32/5 + 8 = 40/5$

$32/5 - 8 = 24/5$

(۵)

$d = \frac{-95}{2} + 24 = \frac{1}{2}$

$a_{101} = a_1 + 100d \rightarrow -24 + \frac{100}{4} = 1$

$t_n = 128 \times r^7 = 1$

$r^7 = \frac{1}{128} \rightarrow r = \frac{1}{2}$

(۶)

$(a_7)^2 = a_3 \times a_9$

$(a_1 + 6d)^2 = (a_1 + 2d)(a_1 + 8d)$

$a_1^2 + 36d^2 + 12a_1 d = a_1^2 + a_1 d + 2da + 16d^2$

$20d^2 + 2a_1 d = 0$

$2d(10d + a) = 0 \rightarrow 2d = 0 \rightarrow d = 0$

یا

$10d + a = 0 \rightarrow 10d = -a$

$d = \frac{-a}{10}$

(۷)

$(a+d)(a+7d) = (a+3d)^2 \rightarrow a^2 + 7d^2 + 8ad = a^2 + 9d^2 + 6ad$

$2ad = 2d^2 \rightarrow a = d$

$2a, 4a, 8a \rightarrow r = \frac{4a}{2a} = 2 \rightarrow a_{10} = a_1 r^9 = \frac{1}{4} \times 2^9 = 2^{-2} \times 2^9 = 2^7 = 128$

(۸)

$3a_2, 2a_3, a_4 \rightarrow 2 \times 2aq^2 - 3aq = aq^3 - 2aq^2 \rightarrow aq^3 - 4aq^2 + 3qa = 0$

$\underset{\neq 0}{aq}(q^2 - 4q + 3) = 0 \rightarrow (q-3)(q-1) = 0$

$q = 3$ ✓ , 1 ✗ (ثابت)

(۹)

$d = \frac{7}{4} - \frac{8}{4} = -\frac{1}{4}$ $a_4 = \frac{5}{4}$, $a_8 = 2 - \frac{7}{4} = \frac{1}{4}$ $a_{12} = 2 - \frac{12}{4} = -1$

$\frac{\frac{1}{4} + x}{\frac{5}{4} + x} = \frac{-1 + x}{\frac{1}{4} + x} \rightarrow (1+4x)^2 = 16x^2 - 20 + 4x \rightarrow -4x = -21 \rightarrow x = -\frac{21}{4}$

$x = -\frac{21}{4} \rightarrow -\frac{16}{4}, -\frac{20}{4}, -\frac{25}{4} \rightarrow r = \frac{-\frac{20}{4}}{-\frac{16}{4}} = \frac{-\frac{25}{4}}{-\frac{20}{4}} = 2 \rightarrow r = \frac{5}{4}$

(۱۰)

$a_1 + a_4 + a_7 = 73 \rightarrow a_1 + ar^3 + ar^6 = 73 \rightarrow a(1 + r^3 + r^6) = 73$

$a_1 = t_1$ $a_2 = t_4 \rightarrow a_1 + d$ $a_7 = t_{10} = a + 9d$

$ar^6 = a_1 + 9(a(r^3 - 1)) \rightarrow$

$a(9r^3 - 8) \rightarrow r^6 = 9r^3 - 8 \rightarrow r^6 - 9r^3 + 8 = 0 \rightarrow \underset{r^3=1}{(r^3 - 1)}\underset{r^3=8}{(r^3 - 8)} = 0$

$r^3 = 1 \rightarrow r = 1$, $r^3 = 8 \rightarrow r = 2$. $d = a(r^3 - 1) \rightarrow d = a(8-1) = d = 7a_1$

$a_1 + a_1 r^3 + a_1 r^6 = 73 \rightarrow a + 8a + 64a \rightarrow 73a = 73$

$a = 1 \rightarrow d = (1) \cdot 7$

!

$r = 1 \leftarrow d = 0$