تکلیف ۱۵ ماندانا رئیس نژاد ۱۸,۷۵

۱)
$\frac{a}{q} \times a \times aq = 64 \Rightarrow a^3 = 64 \longrightarrow a = 4$, $\frac{a}{q} + a + aq = 21 \Rightarrow \frac{4}{q} + 4 + 4q = 21 \xrightarrow{\times q} 4q^2 + 4q + 4 = 21q \Rightarrow 4q^2 - 17q + 4 = 0$

$\Delta = 289 - 64 = 225 \Rightarrow x = \frac{17 \pm \sqrt{225}}{8} \to 4 \Rightarrow \boxed{q = \frac{1}{4}}$

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۲)
$a_1 = x^2+4$, $a_2 = 2x$, $a_3 = x^2-2$, $0<q<1$ (چون نزولی) , $\frac{a_2}{a_1} = \frac{a_3}{a_2} \xrightarrow{\text{جایگذاری}} \frac{2x}{x^2+4} = \frac{x^2-2}{2x}$

$4x^2 = (x^2+4)(x^2-2) \Rightarrow 4x^2 = x^4 + 4x^2 - 2x^2 - 8 \Rightarrow 4x^2 = x^4 + 2x^2 - 8 \Rightarrow x^4 - 2x^2 - 8 = 0$

$x^2 = y \Rightarrow y^2 - 2y - 8 = 0 \to (y-4)(y+2) = 0 \to x^2 = 4 \Rightarrow x = \pm 2$
$2 \to a_1 = 8, a_2 = 4, \boxed{q = \frac{1}{2}}$ ✓ $a_3 = 2$
$-2 \to a_1 = 8, a_2 = -4, a_3 = 2$ ✗

$S_n = a\left(\frac{1-q^7}{1-q}\right) \longrightarrow 8 \times \left(\frac{1-\frac{1}{128}}{1-\frac{1}{2}}\right) = \frac{8 \times \frac{127}{128}}{\frac{1}{2}} = \frac{127 \times 2}{16} = 15.875$ در نتیجه

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۳)
$1 + q + q^2 + q^3 + q^4 = \frac{121}{81} \times a_1 \Rightarrow a_1(1 + q + q^2 + q^3 + q^4) = \frac{121}{81} a_1 \Rightarrow a_1 + aq + aq^2 + aq^3 + aq^4 = \frac{121}{\cancel{81}} \times \cancel{243}^{3} \Rightarrow a_1 + aq + aq^2 + aq^3 + aq^4 = \boxed{363}$

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۴)
$d = \frac{b-a}{n+1} \Rightarrow \frac{64-1}{6} = \boxed{10.5}$, درج وسطی $= a_4 \longrightarrow a_4 = 1 + (3) \times 10.5 = \boxed{32.5}$

$\frac{b}{a} = q^{n+1} \to \frac{64}{1} = q^6 \to q = \pm 2 \longrightarrow$
if $q = -2 \Rightarrow a_4 = 1 \times (-2)^3 = -8$
if $q = 2 \Rightarrow a_4 = 1 \times (2)^3 = +8$ A

$A + B = 32.5 \pm 8 \to \boxed{40.5}$ ✓ , $\boxed{24.5}$ ✓

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۵)
$a_1 + (n-1)d \to -24 + (n-1)\frac{1}{4} \to a_{101} = -24 + \cancel{100}^{25} \times \frac{1}{\cancel{4}} = \boxed{1}$

$d = -\frac{95}{4} = -24 = \frac{1}{4}$

$a_1 \times q^{n-1} \to 128 \times q^7 = 1 \Rightarrow q^7 = \frac{1}{128} \Rightarrow \boxed{q = \frac{1}{2}}$

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۶)
$(a_1 + 8d)(a_1 + 2d) = (a_1 + 4d)^2 \Rightarrow a^2 + 10ad + 16d^2 = a^2 + 12ad + 36d^2 \Rightarrow$
$-20d^2 = 20d^2 \Rightarrow -2a = \cancel{20d} \Rightarrow a = -10d \Rightarrow \boxed{d = -\frac{1}{10}a}$

$d = 0 \longrightarrow$ در صورت تقسیم مهم است

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۷)
$a_1 + d \Rightarrow 2d$, $a_1 + 3d \Rightarrow 4d$, $a_1 + 7d \Rightarrow 8d \Rightarrow (a_1 + 3d)^2 = (a_1 + d)(a_1 + 7d)$

$a^2 + 9d^2 + 6ad = a^2 + 8ad + 7d^2 \Rightarrow 2d^2 = 2ad \Rightarrow \boxed{d = a}$; $q = 2$, $a_1 = \frac{1}{4} \Rightarrow a_{10} = a_1 q^9 \Rightarrow$

$\frac{1}{4} \times 2^9 = \frac{1}{2^2} \times 2^{9} = \boxed{128}$

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۸)
$3a_2 \Rightarrow 3a_1q$, $2a_3 \Rightarrow 2a_1q^2$, $a_4 \Rightarrow a_1q^3 \Rightarrow \frac{3a_1q + a_1q^3}{2} = 2a_1q^2 \Rightarrow 3a_1q + a_1q^3 = 4a_1q^2 \Rightarrow \cancel{a_1}q(3 + q^2) = 4\cancel{a_1}q^{\cancel{2}} \Rightarrow 3 + q^2 = 4q \Rightarrow q^2 - 4q + 3 = 0$ → $1 \to$ ✗ , 3 ✓

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۹)
$d = -\frac{1}{4}$, $a = 2$

(۱) $a + 3d + x \to 1 + 3(-\frac{1}{4}) + x \to -\frac{5}{4} + x \Rightarrow \frac{5}{4} - \frac{21}{4} = -4$

(۲) $a + 7d + x \to \frac{1}{4} + x \to \frac{1}{4} - \frac{21}{4} = -5$

(۳) $a + 12d + x \to -1 + x$

$b^2 = ac \to (\frac{1}{4} + x)^2 = (x-1)(x + \frac{5}{4})$
$x^2 + \frac{1}{16} + \frac{1}{2}x = x^2 + \frac{1}{4}x - \frac{5}{4} \Rightarrow -\frac{21}{16} = \frac{1}{4}x \Rightarrow \boxed{x = -\frac{21}{4}}$

$q^{m-n} = \frac{a_m}{a_n} = q^{2-1} = -\frac{5}{4} \Rightarrow \boxed{q = \frac{5}{4}}$ جواب

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$$a_1 + a_1 q^{۳} + a_1 q^{۶} = ۷۳ \xrightarrow{q=۲} a_1 + ۸a_1 + ۶۴a_1 = ۷۳a_1 = ۷۳ \longrightarrow a_1 = ۱$$ ۱۰)

$$d = \frac{a_۴ - a_1}{۱} = a_1 q^{۳} - a_1 = a_1(q^{۳}-1) \longrightarrow d = a_1(q^{۳}-1) \longrightarrow ۱(۸-۱) = ۷$$ → جواب

$$۹d = a_1 q^{۶} - a_1 = a_1(q^{۶}-1) \Rightarrow d = \frac{a_1(q^{۶}-1)}{۹} \Rightarrow \frac{a_1(q^{۶}-1)}{۹} = a_1(q^{۳}-1)$$

$$\frac{(q^{۳}+1)(q^{۳}-1)}{۹} = q^{۳}-1 \Rightarrow q^{۳}+1 = ۹ \Rightarrow q^{۳} = ۸ \rightarrow \boxed{q = ۲}$$

$$q = 1 \qquad d = aq^{۳} - a$$

$$۳a = ۷۳ \qquad \frac{۷۳}{۳} - \frac{۷۳}{۳} = ۰$$

$$a = \frac{۷۳}{۳}$$

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