عسل رمضانی — دهم دختران A — تکلیف ۱۵

۱

$a_1\times a_2\times a_3 = 64 \Rightarrow \frac{a_2}{q}\times a_2\times a_2q = 64 \Rightarrow a_2 = 4$

$a_1+a_2+a_3 = 21 \Rightarrow \frac{a_2}{q}+a_2+a_2q = 21 \Rightarrow \frac{4}{q}+4+4q = 21 \Rightarrow \frac{4}{q}+4q = 17 \Rightarrow 4+4q^2 = 17q$

$\Rightarrow 4q^2-17q+4 = 0 \Rightarrow q^2-17q+16 = 0 \Rightarrow (q-1)(q-16) = 0 \to q = \frac{16}{4} = 4$ ✗

$\to q = \frac{1}{4}$ ✓

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$a_1\times a_3 = a_2\times a_2 \Rightarrow (x^2-2)(x^2+4) = 4x^2 \Rightarrow x^4+2x^2-8 = 4x^2$

$x^4-2x^2-8 = 0 \Rightarrow (x^2-4)(x^2+2) = 0 \to x^2 = -2$ ✗

$\to x^2 = 4 \Rightarrow x = \pm 2$

$x = 2 \Rightarrow a_n = 8, 4, 2, \dots$ ✓ $\quad S_7 = a\left(\frac{1-q^7}{1-q}\right) = 8\times\left(\frac{1-(\frac{1}{2})^7}{1-\frac{1}{2}}\right) = \frac{8-\frac{8}{2^7}}{\frac{1}{2}}$

$x = -2 \Rightarrow a_n = 8, -4, 2, \dots$ ✗

$= 16-\frac{16}{2^7} = 16-\frac{1}{8} = \boxed{\frac{127}{8}}$

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$S_5 = a+aq+aq^2+aq^3+aq^4 = a(1+q+q^2+q^3+q^4) = 243\times\frac{121}{81} = 3\times 121 = \boxed{363}$

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$d = \frac{64-1}{5+1} = \frac{63}{6} = 10.5 \to A = a_4 = a+3d = 1+31.5 = 32.5$

واسطه هندسی $= t_n = 1, 2, 4, 8, 16, 32, 64 \to B = t_4 = 8 \to A+B = 8+32.5 = \boxed{40.5}$

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$t_n = -24, -\frac{95}{4}, \dots \to d = -\frac{95}{4}-(-24) = -\frac{95}{4}+\frac{96}{4} = \frac{1}{4} \Rightarrow t_{101} = a+100d = -24+25 = \boxed{1}$

$a_n = 128, a_2, \dots, a_8 = 1 \Rightarrow \frac{a_8}{a_1} = \frac{1}{128} = \frac{aq^7}{a} \Rightarrow q = \frac{1}{2}$

۶

$a_3\times a_9 = a_7 \to (a+2d)(a+8d) = (a+6d)^2 \Rightarrow a^2+10ad+16d^2 = a^2+36d^2+12ad$

$\Rightarrow 20d^2+2ad = 0 \to 2d(10d+a) = 0 \to 2d = 0 \Rightarrow d = 0$

$\to 10d+a = 0 \Rightarrow 10d = -a \Rightarrow d = \frac{-a}{10}$

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$a_2\times a_8 = a_4\times a_4 \to (a+d)(a+7d) = (a+3d)^2 \Rightarrow a^2+8ad+7d^2 = a^2+6ad+9d^2 \to 2d^2-2ad = 0$

$\Rightarrow 2d(d-a) = 0 \to d = 0$ ✗

$\to a = d$ ✓

دنباله هندسی $= t_n = \dots a+d, a+3d, a+7d \dots = \dots 2a, 4a, 8a, \dots \Rightarrow q = 2$

$t_{10} = tq^9 = \frac{1}{4}\times 2^9 = 2^7 = 128$

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$a_n = 3aq, 2aq^2, aq^3, \dots \Rightarrow 3aq+aq^3 = 4aq^2 \Rightarrow 3+q^2 = 4q \Rightarrow q^2-4q+3 = 0$

$(q-1)(q-3) = 0 \to q = 1$ ✗

$\to q = 3$ ✓

$a_n = ۲ , \frac{۷}{۴} , \dots \Rightarrow d = \frac{۷}{۴} - ۲ = \frac{-۱}{۴} \Rightarrow a_۴ = a + ۳d = ۲ - \frac{۳}{۴} = +\frac{۵}{۴}$

$a_۸ = a + ۷d = a - \frac{۷}{۴} = \frac{۱}{۴} \qquad a_{۱۳} = a + ۱۲d = ۲ - \frac{۱۲}{۴} = \frac{-۴}{۴}$

دنباله هندسی $= t_n = \frac{۵}{۴} + x , \frac{۱}{۴} + x , -۱ + x , \dots \Rightarrow (x + \frac{۵}{۴})(x - ۱) = (x + \frac{۱}{۴})^۲$ ۹

$\Rightarrow x^۲ + \frac{۱}{۴}x - \frac{۵}{۴} = \frac{۱}{۱۶} + x^۲ + \frac{x}{۲} \Rightarrow \frac{x}{۴} + \frac{۲۱}{۱۶} = ۰ \Rightarrow \frac{x}{۴} = \frac{-۲۱}{۱۶} \Rightarrow x = -\frac{۲۱}{۴}$

$t_n = \frac{-۱۶}{۴} , \frac{-۲۰}{۴} , \frac{-۲۵}{۴} , \dots \Rightarrow \boxed{q = \frac{۵}{۴}}$

$t_۱ \times t_۷ = t_۲ \times t_۲ \Rightarrow a(a + ۹d) = (a + d)^۲ \Rightarrow a^۲ + ۹ad = a^۲ + d^۲ + ۲ad \Rightarrow d^۲ - ۷ad = ۰$

$d(d - ۷a) = ۰ \Rightarrow d = ۰$ | $t_۱ + t_۲ + t_۷ = ۷۳ \Rightarrow a + a + d + a + ۹d = ۷۳$ ۱۰

$\Rightarrow d = ۷a$

$\Rightarrow ۳a + ۱۰d = ۷۳ \Rightarrow ۷۳a = ۷۳ \Rightarrow a = ۱ \Rightarrow \underline{d = ۷}$