۱۹/۷۵

نام و نام خانوادگی ......ماهان اجری...... پاسخنامه تشریحی تکلیف شماره ...۱۸... کلاس ......دهم......A......

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$a_n = \frac{1}{2}, 1, 2, \dots \Rightarrow r=2$

الف) $a_{10}=? \Rightarrow a_{10}=a_1 \cdot r^9 = \frac{1}{2} \times 2^9 = 2^8 = 256$

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ب) $\boxed{\frac{a_{17}}{a_{13}} = \frac{\cancel{a_{13}} \cdot r^4}{\cancel{a_{13}}} = 2^4 = 16 \ / \ \frac{a_{17}}{a_{13}} = \frac{\cancel{a_1} \cdot r^{16}}{\cancel{a_1} \cdot r^{12}} = r^4 = 16}$ $\rightarrow a_7 = a_1 \cdot r^6 = \frac{1}{2} \times 2^{6} = 32$

ج) $\frac{10+4}{2}=7$ (عدد وسط بین ۱۰ و ۴) $\Rightarrow (a_7)^2 = a_4 \cdot a_{10}$ $\begin{cases} a_4 = \frac{1}{2} \times 2^3 = 4 \\ a_{10} = \frac{1}{2} \times 2^9 = 512 \end{cases}$ $\Rightarrow (a_7)^2 = 512 \times 4 = 1024 \Rightarrow a_7 = 32$

د) $a_n = 128 \Rightarrow \underset{\frac{1}{2}}{a_1} \cdot r^{n-1} = 128 \Rightarrow 2^{n-1} = 128(2) = 256 \Rightarrow 2^{n-1} = 2^8$

$n-1 = 8 \Rightarrow n = 9$, $a_9 = 128$

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$\left.\begin{matrix} a_5 = 12 \\ a_8 = 96 \end{matrix}\right\}$ $\frac{a_8}{a_5} = \frac{\cancel{a_1} \cdot r^{7}}{\cancel{a_1} \cdot r^4} = \frac{96}{12} \Rightarrow r^3 = 8 \Rightarrow r=2$, $a_1 \cdot \underbrace{r^4}_{2^4} = 12 \Rightarrow a_1 = \frac{12}{16} = \frac{3}{4}$

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$a_{10} = ? \Rightarrow a_1 \cdot r^9 = \frac{3}{\cancel{4}} \times 2^{9} = 3 \times \underbrace{2^7}_{128} = 384$

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$a_1 \times \overbrace{a_2 \times a_3 \times a_4}^{a_3 \times a_3} \times a_5 = \underbrace{243}_{3^5} \Rightarrow (a_3)^5 = 3^5 , a_3 = 3 \Rightarrow$ قسمت الف

الف) $\underbrace{a_3}_{a_1 \times r^2} \Rightarrow$ عبارت بالا $= a_1^5 \times r^{10} = 3^5 \xrightarrow{\text{ریشه پنجم}} \sqrt[5]{a_1^5 \times r^{10}} = \sqrt[5]{3^5} = \boxed{a_1 \cdot r^2 = 3}$

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ب) $a_1 \cdot a_5 = a_3^2$

$a_1 \times a_2 \times \underset{3}{\cancel{a_3}} \times a_4 \times a_5 = 243 \Rightarrow (a_1 \times a_5)^2 = \underset{9^2}{81} \xrightarrow{\text{جذر}} \boxed{a_1 \times a_5 = 9}$

* $a_1 \times a_5 = a_2 \times a_4$

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$2^a , 4\sqrt{2} , 2^b \Rightarrow b^2 = ac \Rightarrow (4\sqrt{2})^2 = 2^a \times 2^b \Rightarrow 32 = 2^{a+b} \Rightarrow 2^5 = 2^{a+b} \Rightarrow$

$a+b = 5$

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واسطه حسابی بین a, b $\Rightarrow \frac{a+b}{2} = \frac{5}{2} = \boxed{2/5}$

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$a_3 = 1-x$, $\underbrace{a_7 = x}_{\text{واسطه ۱۱ و ۳}}$, $a_{11} = x+1 \Rightarrow (a_7)^2 = a_3 \times a_{11}$

$\frac{11+3}{2} = 7$

$x^2 = (1-x)(x+1) \Rightarrow x^2 = 1 - x^2$

$2x^2 = 1$

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$x^2 = \frac{1}{2} \Rightarrow$

$x = \sqrt{\pm \frac{1}{2}}$

اینطوری درسته: $\pm \frac{\sqrt{2}}{2}$ و $-\frac{\sqrt{2}}{2}$ حتما مورد قبول نیست

۶	$a_1 + a_4 = 28 \rightsquigarrow a_1 + a_1 q^3 = 28 \rightsquigarrow a_1(1+q^3) = 28$ * $a_2 + a_3 = 12 \rightsquigarrow a_1 q + a_1 q^2 = 12 \rightsquigarrow a_1 q(q+1) = 12 \rightsquigarrow a_1(q+1) = \frac{12}{q}$ ☆ $\hookrightarrow a_1+a_2+a_3+a_4 = 40 \rightsquigarrow S_4 = a_1\left(\frac{q^4-1}{q-1}\right) = 40$ $a_1\left(\frac{(q^2+1)(q^2-1)}{q-1}\right)$, $q^2-1 \to (q-1)(q+1)$ $\Rightarrow a_1\left(\frac{(q^2+1)\cancel{(q-1)}(q+1)}{\cancel{(q-1)}}\right) = 40 \rightsquigarrow$ ☆ $a_1(q+1)(q^2+1) = 40$ $\frac{12}{q}(q^2+1) = 40$ $\left(\frac{12}{q} + 12q = 40\right) \times q \rightsquigarrow 12 + 12q^2 = 40q \rightsquigarrow (12q^2 - 40q + 12 = 0) \div 4$ $3q^2 - 10q + 3 = 0 \rightsquigarrow q^2 - 10q + 9 = 0$, $(q-1)(q-9) = 0$ $\begin{cases} q = \frac{1}{3} \\ q = \frac{9}{3} = 3 \end{cases}$ * $\rightsquigarrow \begin{cases} \text{if } q = \frac{1}{3}: a_1\left(1 + \frac{1}{27}\right) = 28 \rightsquigarrow a_1 = \frac{28}{\frac{28}{27}} = 27 \\ \text{if } q = 3: a_1(1+27) = 28 \rightsquigarrow a_1 = 1 \end{cases}$ بزرگترین جمله $q = \frac{1}{3} \to a_1$: $27, 9, 3, 1$ بزرگترین جمله $q = 3 \to a_4$: $1, 3, 9, 27$ ۳
۷	$a_1 + \underbrace{a_2}_{a_1 r} + \underbrace{a_3}_{a_1 r^2} = 13 \rightsquigarrow a_1(1+r+r^2) = 13$ ☆ $a_1 \times \underbrace{a_2}_{a_1 r} \times \underbrace{a_3}_{a_1 r^2} = \underbrace{27}_{3^3} \rightsquigarrow a_1^3 \cdot r^3 = 3^3 \rightsquigarrow \sqrt[3]{a_1^3 \cdot r^3 = 3^3} = \boxed{a_1 \cdot r = 3} \rightsquigarrow a_2$ $\left[a_1 = \frac{a_1 r}{r} = \frac{3}{r}\right] \rightsquigarrow$ ☆ $a_1(1+r+r^2) = 13 \rightsquigarrow \times r\left[\frac{3}{r}(1+r+r^2) = 13\right] \rightsquigarrow$ $3(1+r+r^2) = 13r \rightsquigarrow 3r^2 - 13r + 3r + 3 = 0 \rightsquigarrow 3r^2 - 10r + 3 = 0 \rightsquigarrow r^2 - 10r + 9 = 0$, $(r-1)(r-9) = 0$ if $r = 3 \to 1, 3, 9$ if $r = \frac{1}{3} \to 9, 3, 1$ $\Rightarrow \begin{cases} a_1, a_2, a_3 \\ 1, 3, 9 \to r = 3 \\ 9, 3, 1 \to r = \frac{1}{3} \end{cases} \Longleftarrow \begin{cases} r = \frac{1}{3} \\ r = \frac{9}{3} = 3 \end{cases}$ ۳
۸	$a_n = 2, 6, 18, \dots \rightsquigarrow r = 3 \Rightarrow \frac{6}{2} = 3$ الف) $S_{10} \Rightarrow S_n = a_1\left(\frac{q^n-1}{q-1}\right) \rightsquigarrow S_{10} = 2\left(\frac{3^{10}-1}{3-1}\right) = 3^{10} - 1 = 59048$ ب) $(a_1 \times a_2 \times \dots \times a_{10}) \rightsquigarrow a_1^n \cdot r^{\frac{n(n-1)}{2}} \rightsquigarrow 2^{10} \times 3^{\frac{10(9)}{2}} \rightsquigarrow \underbrace{2^{10}}_{1024} \times 3^{45} = 1024(3^{45})$ ۳
۹	$b_n = a_{n+1} - a_n = a_1 \underbrace{q^n}_{q^{n+1-1}} - a_1 q^{n-1} = a_1 q^{n-1}(q-1) \rightsquigarrow$ b_n هندسی است $\frac{b_{n+1}}{b_n} = \frac{\cancel{a_1} q^{n+1-1}\cancel{(q-1)}}{\cancel{a_1} q^{n-1}\cancel{(q-1)}} = \frac{q^n}{q^{n-1}} \quad q^{n-n+1} = q^1 = q$ ۳
۱۰	الف) دنباله حسابی: $a_1, a_1+d, a_1+2d, \dots, a_1+(n-1)d$ جمع n جمله = $\begin{cases} S_n = a_1 + a_1+d + \dots + a_1+(n-1)d \\ S_n = a_1+(n-1)d + a_1+(n-2)d + \dots + a_1 \end{cases}$ + (دو رابطه را جمع می‌کنیم) معکوس جمع n جمله = $2S_n = n(2a_1 + (n-1)d) \rightsquigarrow$ $\boxed{S_n = \frac{n}{2}(2a_1 + (n-1)d)}$ ب) دنباله هندسی: $a_1, a_1 q, a_1 q^2, \dots, a_1 q^{(n-1)}$ جمع n جمله = , $S_n = a_1 + a_1 q + a_1 q^2 + \dots + a_1 q^{n-1}$ $\xrightarrow[\text{ضرب در } q]{} q S_n = a_1 q + a_1 q^2 + \dots + a_1 q^n$ − (دو رابطه را از هم کم می‌کنیم) $q S_n - S_n = a q^n - a$ $\hookrightarrow S_n(q-1) = a(q^n - 1)$ $= \boxed{S_n = a\left(\frac{q^n-1}{q-1}\right)}$ ۳