

$$a_n = \frac{1}{p}, 1, 2, \dots$$

الف) $a_n = 2^{n-2} \Rightarrow a_{10} = 2^{10-2} = 2^8 = 256$

ب) $\frac{a_{14}}{a_{13}} = \frac{a_1 q^{14}}{a_1 q^{13}} = q^1 = 2^1 = 2$

ج) $a_5 = 2^{5-2} = 2^3 = 8$

د) $2^{n-2} = 128 \Rightarrow 2^{n-2} = 2^7 \Rightarrow n-2=7 \Rightarrow n=9$

$$\frac{a_1}{a_8} = \frac{a_1 q^1}{a_1 q^8} = q^{-7} = \frac{1}{128} = 2^{-7} \Rightarrow q = 2 \Rightarrow a_{10} = a_1 q^9 = 1 \times 2^9 = 512$$

الف) $a_1 \times a_2 \times a_3 \times a_4 \times a_5 = 2^4 \times 2^8 \Rightarrow \frac{a_1}{q^4} \times \frac{a_1}{q^3} \times \frac{a_1}{q^2} \times \frac{a_1}{q^1} \times a_1 q^0 = a_1^5 = 2^{12} \Rightarrow a_1 = 2^3$

ب) $a_1 \times a_5 = \frac{a_1}{q^4} \times a_1 q^4 = a_1^2 = 2^4 = 4$

$2^a, \sqrt[4]{2}, 2^b \Rightarrow (\sqrt[4]{2})^r = 2^a \times 2^b \Rightarrow 2^{\frac{r}{4}} = 2^{a+b} \Rightarrow \frac{r}{4} = a+b \Rightarrow a+b = \frac{r}{4}$

واسطه حسابی $\Rightarrow \frac{a+b}{2} = \frac{r}{4} = \frac{1}{2} \Rightarrow a+b = 1$

۱- $x, x, x+1 \Rightarrow x^r = (x+1)(1-x) = 1-x^2 \Rightarrow 2x^r = 1 \Rightarrow x^r = \frac{1}{2} \Rightarrow x = \pm \sqrt[4]{\frac{1}{2}}$

$a_1 + a_4 = 18 \Rightarrow a_1 + a_1 q^3 = 18$

$a_2 + a_5 = 12 \Rightarrow a_1 q + a_1 q^4 = 12 \Rightarrow a_1 q(q+1) = 12 \Rightarrow a_1(q+1) = \frac{12}{q}$

$a_1 \left(\frac{q^4-1}{q-1} \right) = 18 \Rightarrow a_1 \left(\frac{(q^2+1)(q-1)(q+1)}{q-1} \right) = a_1(q^2+1)(q+1) = 18 \Rightarrow \frac{12}{q}(q^2+1) = 18$

$\Rightarrow 12q + \frac{12}{q} = 18 \Rightarrow 12q^2 + 12 = 18q \Rightarrow 12q^2 - 6q + 12 = 0 \Rightarrow 2q^2 - 10q + 2 = 0 \Rightarrow$

$\Delta = 100 - 16 = 84 \Rightarrow \frac{10 \pm \sqrt{84}}{4} = \frac{5 \pm \sqrt{21}}{2} \Rightarrow q = \frac{1}{2} \Rightarrow a_1 = 27$ بزرگترین جمله

$\frac{a_1}{q^3} \left(\frac{1}{q} \right) = 12 \Rightarrow a_1 = 27$

$q = \frac{1}{2} \Rightarrow a_1 \times 4 = 12 \Rightarrow a_1 = 3 \Rightarrow a_4 = 3 \times 2^3 = 24$

$\Rightarrow a_4 = 24$ بزرگترین جمله

$a_1 + a_2 + a_3 = 13$

$a_1 + a_3 = 10 \Rightarrow a_1 + a_1 q^2 = 10 \Rightarrow \frac{13}{q} + \frac{13}{q} \times q^2 = \frac{13}{q} + 13q = 10$

$a_1 \times a_2 \times a_3 = 27 \Rightarrow \frac{a_1}{q} \times a_1 \times a_1 q = a_1^3 = 27 \Rightarrow a_1 = 3$

$a_1 q = 3 \Rightarrow a_1 = \frac{3}{q}$

$\frac{13}{q} + 13q = 10 \Rightarrow 13q^2 + 13 = 10q \Rightarrow 13q^2 - 10q + 13 = 0$

$\Delta = 100 - 169 = -69 \Rightarrow \frac{10 \pm \sqrt{-69}}{26} = \frac{5 \pm \sqrt{-69}}{13}$ $q = 3 \Rightarrow \frac{a_1}{3} = 1 \Rightarrow a_1 = 3$

$q = 3 \Rightarrow 1, 3, 9$
 $q = \frac{1}{3} \Rightarrow 9, 3, 1$

$q = \frac{1}{3} \Rightarrow 10q = 10 \times \frac{1}{3} = \frac{10}{3} \Rightarrow a_1 = 9$

$$a_n = 2, 4, 18, \dots$$

$$\text{الف) } S_{10} = 2 \left(\frac{3^{10} - 1}{3 - 1} \right) = 2 \left(\frac{3^{10} - 1}{2} \right) = 3^{10} - 1$$

$$\text{ب) } P_{10} = (2, 2, 3^9)^{\Delta} = 10 \times 3^9 = 10 \times 10^3 = 10^4 \times 10^3 = 10^7$$

$$a_1, a, q, a, q^2, a, q^3, \dots$$

$$a, q - a_1, a, q^2 - a, q, a, q^3 - a, q^2$$

$$a_1(q-1), a_1 q(q-1), a_1 q^2(q-1) \quad \text{دنباله هندسی}$$

$$\frac{a_1 q(q-1)}{a_1(q-1)} = \frac{a_1 q^2(q-1)}{a_1 q(q-1)} = q$$

$$\text{الف) } a_1 + (a_1 + d) + (a_1 + 2d) + \dots + (a_1 + (n-1)d) = na_1 + \frac{n(n-1)d}{2} = \frac{n}{2} (2a_1 + (n-1)d)$$

$$\text{ب) } S_n = a + a, q + a, q^2 + \dots + a, q^{n-1}$$

$$qS_n = a, q + a, q^2 + a, q^3 + \dots + a, q^n$$

$$qS_n - S_n = a, q^n - a_1 \Rightarrow S_n(q-1) = a_1(q^n - 1) \Rightarrow S_n = a_1 \left(\frac{q^n - 1}{q - 1} \right)$$