

شماره ۱۴

- (۱) $a_n = \frac{1}{2}, 1, 2, \dots$ $q=2$ $a_1 = \frac{1}{2}$ $a_n = a_1 \cdot q^{n-1} \rightarrow \frac{1}{2} \cdot 2^{n-1}$ $a_{10} = \frac{1}{2} \cdot 2^9 \rightarrow 2^9$ (الف)
- (ب) $\frac{a_{14}}{a_{13}} = \frac{a_1 q^{14}}{a_1 q^{13}} = q = 2$ $a_{14} = 2^{14}$
- (ج) رابطه‌ی عمومی بین ضلعات ۴ و ۵
 $b^2 = ac \rightarrow 2^2 \cdot 2^8 = 2^2 \cdot b^2 / 4 \rightarrow \frac{1}{4} \times 2^8 = b^2$
 $b = 2^4 = 16$
- (د) $128, 2^7 \rightarrow a_1 \cdot q^{n-1} \rightarrow \frac{1}{2} \cdot 2^{n-1}$
 $n-1 = 7 \rightarrow n = 8$
- (۲) $a_5 = 12$ $a_{10} = ?$ $\frac{a_n}{a_5} = \frac{a_1 q^n}{a_1 q^5} = q^5 = \frac{94}{12} = q^5 = 1 \rightarrow q = 2$
 $a_5 = a_1 \cdot q^{n-1} \rightarrow a_1 \cdot q^4 = 12 \rightarrow a_1 = \frac{12}{16} = \frac{3}{4}$
 $a_{10} = a_1 \cdot q^9 \rightarrow \frac{3}{4} \times 2^9 = 3 \times 2^7 = 384$
- (۳) $a_1 \times a_2 \times a_3 \times a_4 \times a_5 = 243$
 $P_n = (a_1 \times a_n)^n$ $P_n = \sqrt{(a_1 \times a_n)^n} \rightarrow P_n = \sqrt{(a_1 \times a_n)^2} = \frac{243}{3^2} \rightarrow (a_1 \times a_n)^2 = (3^5)^2 \cdot (3^2)^2$
 $P_n = (a_1 \times a_n)^2 = 243 \rightarrow a_1 \times a_n = 3$
- (۴) $r^a, \sqrt[r]{r}, r^b$
 $\sqrt[r]{r} = r^{\frac{1}{r}} \rightarrow r^{\frac{1}{r}}$
 $r^a, r^{\frac{1}{r}}, r^b$ رابطه‌ی هندسی
 $b^2 = ac \rightarrow (\frac{r^{\frac{1}{r}}}{r^a})^2 = r^a \cdot r^b$
 $(\frac{1}{r^a})^2 = \frac{r^a}{r^a} \times r^b \rightarrow a+b = a$
- (۵) $a_1 = 1, a$ $b^2 = ac \rightarrow a^2 = (1+a)(1-a)$
 $a_2 = a$ b $a^2 = 1 - a^2 \rightarrow 2a^2 = 1$
 $a_3 = a + 1$ c $a^2 = \frac{1}{2} \rightarrow a = \pm \frac{1}{\sqrt{2}} \rightarrow a = \pm \frac{\sqrt{2}}{2}$
 $\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
- (۶) $a_1 + a_5 = 24 \rightarrow a_1 + a_1 q^4 = 24 \rightarrow a_1(1+q^4) = 24$
 $a_2 + a_4 = 12 \rightarrow a_1 q + a_1 q^3 = 12 \rightarrow a_1 q(1+q^2) = 12$
 $\frac{24}{12} = \frac{a_1(1+q^4)}{a_1 q(1+q^2)} = \frac{(1+q^4)(1-q+q^2)}{q(1+q^2)} \cdot \frac{1}{q} \rightarrow 2 - 2q + 2q^2 = 12q \rightarrow 2 - 10q + 2q^2 = 0$
 $a_1(1+q^4) = 24 \rightarrow a_1(24) = 24$
 $a_1 = 1$
 $(1) \quad (2) \quad (3) \quad (4) \quad \frac{27}{5}$
- (۷) $\frac{a}{q} + a + aq = 3$ $\rightarrow \frac{3}{q} + 3 + 3q = 12 \rightarrow 3 + 3q + 3q^2 = 12q \rightarrow 3q^2 - 10q + 3 = 0 \rightarrow q = 3$
 $\frac{a}{3} \times a \times 3q = a^2 = 27 \rightarrow a = 3$ $q = 3 \rightarrow a_1 = \frac{3}{3} = 1 / a_2 = 3 / a_3 = 9$
 $q = \frac{1}{3} \rightarrow a_1 = 9 / a_2 = 3 / a_3 = 1$
- (۸) $a_n = 2, 6, 18, \dots$ $q = 3$ $a_1 = 2$
 $s_n = a_1 (\frac{q^n - 1}{q - 1}) \rightarrow s_{10} = 2 (\frac{3^{10} - 1}{3 - 1}) = 59049 - 1 = 59048$
 $P_n = \sqrt{(a_1 \cdot a_n)^n} \rightarrow P_{10} = \sqrt{(a_1 \cdot a_{10})^{10}} \rightarrow (2 \times 59048)^{10} \rightarrow (118096)^{10}$
 $a_1 = 2$
 $a_{10} = a_1 q^9 \rightarrow 2 \times 3^9 = 118098 = 118098$

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$$a_1, a_1q, a_1q^2, a_1q^3, \dots$$

$$q \neq 1$$

$$a_1q - a_1 = a_1(q-1) \quad \times q$$

$$a_1q^2 - a_1q = a_1q(q-1) \quad \times q$$

$$a_1q^3 - a_1q^2 = a_1q^2(q-1)$$

$$a_1, a_2, a_3, \dots, a_n$$

$$a_n = a_1 + (n-1)d$$

$$s_n = a_1 + a_2 + a_3 + \dots + a_n$$

$$s_n = a_n + a_{n-1} + a_{n-2} + \dots + a_1$$

$$2s_n = (a_1 + a_n) + (a_2 + a_{n-1}) + \dots + (a_n + a_1)$$

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$$2s_n = n(a_1 + a_n) \rightarrow s_n = \frac{n(a_1 + a_n)}{2} \rightarrow \frac{n(a_1 + a_1 + (n-1)d)}{2}$$

$$s_n = \frac{n}{2}(2a_1 + (n-1)d)$$

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$$a, aq, aq^2, \dots, aq^{n-1}$$

$$s_n = a + aq + aq^2 + \dots + aq^{n-1}$$

$$\times q \rightarrow qs_n = aq + aq^2 + aq^3 + \dots + aq^n$$

$$s_n - qs_n = a - aq^n \rightarrow s_n(1-q) = a(1-q^n) \rightarrow s_n = a \frac{(1-q^n)}{1-q}$$