بنام خدا

زهرا سهرابی — ۲۰ — شاوری تکلیف ۱۴

۱)

$a_n = \frac{1}{2}, 1, 2, \dots$ (×۲ ×۲) $\quad q=2 \quad a_1 = \frac{1}{2} \quad a_n = a_1 q^{n-1} \to \frac{1}{2}\cdot 2^{n-1} \quad a_{10} = \frac{1}{2}\cdot 2^{9} \to 2^{8}$

الف) 2^8

ب) $\frac{a_{17}}{a_{13}} = \frac{a_1 q^{16}}{a_1 q^{12}} = q^4 = 2^4 = 16$

ج) واسطه‌ی هندسی بین جملات ۴ و ۱۰

$b^2 = ac \to 2^2 \cdot 2^8 = 2^{10} = b^2$ / جمله ۴ $= \frac{1}{2} \times 2^3 = 4$ جمله ۱۰ $= \frac{1}{2}\times 2^9 = 2^8$

$\hookrightarrow b = \pm 2^5 \to$ منفی نمی‌تواند $\to b = 32$

د) $128 = 2^7 \to a_1 \times q^{n-1} \to \frac{1}{2} \times 2^{n-1}$ $\quad n-1 = 8 \to n = 9$

۲)

$a_5 = 12 \quad a_{10} = ?$

$a_n = 96$

$\frac{a_8}{a_5} = \frac{a_1 \cdot q^7}{a_1 \cdot q^4} \to q^3 = \frac{96}{12} = q^3 = 8 \to q = 2$

$a_5 = a_1 \cdot q^{n-1} \to a_1 \cdot q^4 = 12 \to a_1 = \frac{12}{16} = \frac{3}{4}$

$a_{10} = a_1 \cdot q^9 \to \frac{3}{4} \times 2^9 = 3 \times 2^7 = 384$

۳)

$a_1 \times a_2 \times a_3 \times a_4 \times a_5 = 243$

$P_n = ($جمله اول، جمله آخر$)^n$

$P_n = \sqrt{(a_1 \times a_n)^n} \to P_5 = \sqrt{(a_1 \times a_5)^5} = 243 \to (a_1 \times a_5)^5 = (3^5)^2 = (3^2)^5$

3^5

$(a_1 \times a_5) = 3^2 \to 9$

$P_5 = (a_3)^5 = 243 \to a_3 = 3$

۴)

$2^a, 4\sqrt{2}, 2^b$

$\sqrt{32} = \sqrt{2^5} \to 2^{\frac{5}{2}}$

$2^a, 2^{\frac{5}{2}}, 2^b$ واسطه‌ی هندسی $b^2 = ac \to (2^{\frac{5}{2}})^2 = 2^a \cdot 2^b$

$(4\sqrt{2})^2 = 32 = 2^a \times 2^b \to a+b = 5$

2^5

واسطه‌ی حسابی a, b $= \frac{a+b}{2} = \frac{5}{2} = ۲٫۵$

۵)

$a_3 = 1 - x \quad a$

$a_7 = x \quad b$

$a_{11} = x+1 \quad c$

$b^2 = ac \to x^2 = (1+x)(1-x)$

$x^2 = 1 - x^2 \to 2x^2 = 1$

$x^2 = \frac{1}{2} \to x = \pm \frac{1}{\sqrt{2}} \to x = \pm\frac{\sqrt{2}}{2}$

$\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

۶)

$a_1 + a_4 = 28 \to a_1 + a_1 \times q^3 = 28 \to a_1(1+q^3) = 28$

$a_2 + a_3 = 12 \to a_1 \times q + a_1 \times q^2 = 12 \to a_1 q(1+q) = 12$

$\frac{28}{12} = \frac{a_1(1+q^3)}{a_1 q(1+q)} = \frac{(1+q)(1-q+q^2)}{q(1+q)} = \frac{7}{3} \to 3 - 3q + 3q^2 = 7q \to 3 - 10q + 3q^2 = 0$ → $q = 3$ → $q = \frac{1}{3}$

$a_1(1+q^3) = 28 \to a_1(28) = 28$

$a_1 = 1$

(۱) 1 (۲) 3 (۳) 9 (۴) 27 بزرگترین جمله $= 27$

۷)

$\frac{a}{q} + a + aq = 13 \to \frac{3}{q} + 3 + 3q = 13 \xrightarrow{\times q} 3 + 3q + 3q^2 = 13q \to 3q^2 - 10q + 3 = 0 \to q = 3$ ، $q = \frac{1}{3}$

$\frac{a}{q} \times a \times aq = a^3 = 27 \to a = 3$

$q = 3 \to a_1 = \frac{3}{3} = 1 \ / \ a_2 = 3 \ / \ a_3 = 3 \times 3 = 9$

$q = \frac{1}{3} \to a_1 = 9 \ / \ a_2 = 3 \ / \ a_3 = 1$

۸)

$a_n = 2, 6, 18, \dots \quad q = 3 \quad a_1 = 2$

الف) $S_n = a_1\left(\frac{q^n - 1}{q-1}\right) \to S_{10} = 2\left(\frac{3^{10}-1}{3-1}\right) = 59049 - 1 = 59048$

ب) $P_n = \sqrt{(a_1 \cdot a_n)^n} \to P_{10} = \sqrt{(a_1 \cdot a_{10})^{10}} \to (2 \times 39366)^5 \to (78732)^5$

$a_1 = 2$

$a_{10} \to a_1 q^9 \to 2 \times 3^9 \to 19683 \times 2 = 39366$

۹)

$a_1, a_1q, a_1q^2, a_1q^3, \dots$

$a_1q - a_1 = a_1(q-1)$ $\times q$

$a_1q^2 - a_1q = a_1q(q-1)$ $\times q^2$

$a_1q^3 - a_1q^2 = a_1q^2(q-1)$

$q = q$

۵

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۱۰) الف)

$a_1, a_2, a_3, \dots, a_n$

$a_n = a_1 + (n-1)d$

$S_n = a_1 + a_2 + a_3 + \dots + a_n$

$+ \quad S_n = a_n + a_{n-1} + a_{n-2} + \dots + a_1$

$2S_n = (a_1 + a_n) + (a_2 + a_{n-1}) + \dots$

$a_1 + a_n$

$2S_n = n(a_1 + a_n) \rightarrow S_n = \frac{n(a_1 + a_n)}{2} \rightarrow \frac{n(a_1 + a_1 + (n-1)d)}{2}$

$S_n = \frac{n}{2}(2a_1 + (n-1)d)$

۵

ب)

$a, aq, aq^2, \dots, aq^{n-1}$

$S_n = a + aq + aq^2 + \dots + aq^{n-1}$

$\times q \rightarrow qS_n = aq + aq^2 + aq^3 + \dots + aq^n$

$qS_n - S_n = aq^n - a \rightarrow S_n(q-1) = a(q^n - 1) \rightarrow S_n = \frac{a(q^n - 1)}{q-1}$