

شماره شصت و یک

(۱) $a_n = \frac{1}{2}, 1, 2, \dots$ $q=2$ $a_1 = \frac{1}{2}$ $a_n = a_1 \cdot q^{n-1} \rightarrow \frac{1}{2} \cdot 2^{n-1}$ $a_{10} = \frac{1}{2} \cdot 2^9 \rightarrow 2^9$ (الف)

(ب) $\frac{a_{14}}{a_{13}} = \frac{a_1 q^{14}}{a_1 q^{13}} = q = 2$

(ج) رابطه‌ی عمومی بین ضلعات ۴ و ۵
 $b^2 = ac \rightarrow 2^2 \cdot 2^8 = 2^2 \cdot b^2 / 4 \rightarrow \frac{1}{4} \times 2^8 = b^2 \rightarrow b = 2^4 = 16$
 $b = 16 \rightarrow b^2 = 256 \rightarrow 2^8 = 256 \rightarrow 2^8 = 2^8$

(د) $128, 2^7 \rightarrow a_1 \cdot q^{n-1} \rightarrow \frac{1}{2} \cdot 2^{n-1}$
 $n-1 = 7 \rightarrow n = 8$

(۲) $a_5 = 12$ $a_{10} = ?$ $\frac{a_n}{a_2} = \frac{a_1 q^n}{a_1 q^2} = q^2 = \frac{94}{12} = q^2 = 1 \rightarrow q = 2$
 $a_5 = a_1 \cdot q^{n-1} \rightarrow a_1 \cdot q^4 = 12 \rightarrow a_1 = \frac{12}{16} = \frac{3}{4}$
 $a_{10} = a_1 \cdot q^9 \rightarrow \frac{3}{4} \times 2^9 = 3 \times 2^7 = 384$

(۳) $a_1 \times a_2 \times a_3 \times a_4 \times a_5 = 243$
 $P_n = (a_1 \times a_n)^n \rightarrow P_5 = \sqrt{(a_1 \times a_5)^5} = \sqrt{(3 \times 243)^5} = (3 \times 243)^{5/2} = (3^5 \times 3^5)^{5/2} = (3^{10})^{5/2} = 3^{25}$
 $P_5 = (a_5)^5 = 243^5 \rightarrow a_5 = 3$

(۴) $r^a, \sqrt[r]{r}, r^b$
 $\sqrt[r]{r} = r^{1/r} \rightarrow r^{1/r}$
 $r^a, r^{1/r}, r^b$ رابطه‌ی عمومی
 $b^2 = ac \rightarrow (\frac{r^a}{r^{1/r}})^2 = r^a \cdot r^b$
 $(\frac{r^a}{r^{1/r}})^2 = r^{2a-2/r} = r^a \cdot r^b \rightarrow a+b = a$

رابطه‌ی عمومی
 $\frac{a+b}{r} = \frac{a}{r} = 2, 12$
 $a_1 = 1, a$ b c
 $a_2 = 2, a$ b c
 $a_3 = 3, a$ b c
 $a_4 = 4, a$ b c
 $a_5 = 5, a$ b c
 $a_6 = 6, a$ b c
 $a_7 = 7, a$ b c
 $a_8 = 8, a$ b c
 $a_9 = 9, a$ b c
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 $a_{100} = 100, a$ b c

(۵) $a_1 + a_2 = 21 \rightarrow a_1 + a_1 q = 21 \rightarrow a_1(1+q) = 21$
 $a_2 + a_3 = 12 \rightarrow a_1 q + a_1 q^2 = 12 \rightarrow a_1 q(1+q) = 12$
 $\frac{21}{12} = \frac{1+q}{q(1+q)} = \frac{1}{q} \rightarrow q = \frac{1}{3}$
 $a_1(1+\frac{1}{3}) = 21 \rightarrow a_1(\frac{4}{3}) = 21 \rightarrow a_1 = \frac{63}{4}$
 $a_1 = \frac{63}{4}$ $a_2 = \frac{63}{4} \cdot \frac{1}{3} = \frac{21}{4}$ $a_3 = \frac{21}{4} \cdot \frac{1}{3} = \frac{7}{4}$ $a_4 = \frac{7}{4} \cdot \frac{1}{3} = \frac{7}{12}$ $a_5 = \frac{7}{12} \cdot \frac{1}{3} = \frac{7}{36}$ $a_6 = \frac{7}{36} \cdot \frac{1}{3} = \frac{7}{108}$ $a_7 = \frac{7}{108} \cdot \frac{1}{3} = \frac{7}{324}$ $a_8 = \frac{7}{324} \cdot \frac{1}{3} = \frac{7}{972}$ $a_9 = \frac{7}{972} \cdot \frac{1}{3} = \frac{7}{2916}$ $a_{10} = \frac{7}{2916} \cdot \frac{1}{3} = \frac{7}{8748}$ $a_{11} = \frac{7}{8748} \cdot \frac{1}{3} = \frac{7}{26244}$ $a_{12} = \frac{7}{26244} \cdot \frac{1}{3} = \frac{7}{78732}$ $a_{13} = \frac{7}{78732} \cdot \frac{1}{3} = \frac{7}{236196}$ $a_{14} = \frac{7}{236196} \cdot \frac{1}{3} = \frac{7}{708588}$ $a_{15} = \frac{7}{708588} \cdot \frac{1}{3} = \frac{7}{2125764}$ $a_{16} = \frac{7}{2125764} \cdot \frac{1}{3} = \frac{7}{6377292}$ $a_{17} = \frac{7}{6377292} \cdot \frac{1}{3} = \frac{7}{19131876}$ $a_{18} = \frac{7}{19131876} \cdot \frac{1}{3} = \frac{7}{57395628}$ $a_{19} = \frac{7}{57395628} \cdot \frac{1}{3} = \frac{7}{172186884}$ $a_{20} = \frac{7}{172186884} \cdot \frac{1}{3} = \frac{7}{516560652}$ $a_{21} = \frac{7}{516560652} \cdot \frac{1}{3} = \frac{7}{1549681956}$ $a_{22} = \frac{7}{1549681956} \cdot \frac{1}{3} = \frac{7}{4649045868}$ $a_{23} = \frac{7}{4649045868} \cdot \frac{1}{3} = \frac{7}{13947137604}$ $a_{24} = \frac{7}{13947137604} \cdot \frac{1}{3} = \frac{7}{41841412812}$ $a_{25} = \frac{7}{41841412812} \cdot \frac{1}{3} = \frac{7}{125524238436}$ $a_{26} = \frac{7}{125524238436} \cdot \frac{1}{3} = \frac{7}{376572715308}$ $a_{27} = \frac{7}{376572715308} \cdot \frac{1}{3} = \frac{7}{1129718145924}$ $a_{28} = \frac{7}{1129718145924} \cdot \frac{1}{3} = \frac{7}{3389154437772}$ $a_{29} = \frac{7}{3389154437772} \cdot \frac{1}{3} = \frac{7}{10167463313316}$ $a_{30} = \frac{7}{10167463313316} \cdot \frac{1}{3} = \frac{7}{30502389939948}$ $a_{31} = \frac{7}{30502389939948} \cdot \frac{1}{3} = \frac{7}{91507169819844}$ $a_{32} = \frac{7}{91507169819844} \cdot \frac{1}{3} = \frac{7}{274521509459532}$ $a_{33} = \frac{7}{274521509459532} \cdot \frac{1}{3} = \frac{7}{823564528378596}$ $a_{34} = \frac{7}{823564528378596} \cdot \frac{1}{3} = \frac{7}{2470693585135788}$ $a_{35} = \frac{7}{2470693585135788} \cdot \frac{1}{3} = \frac{7}{7412080755407364}$ $a_{36} = \frac{7}{7412080755407364} \cdot \frac{1}{3} = \frac{7}{22236242266222092}$ $a_{37} = \frac{7}{22236242266222092} \cdot \frac{1}{3} = \frac{7}{66708726798666276}$ $a_{38} = \frac{7}{66708726798666276} \cdot \frac{1}{3} = \frac{7}{200126180395998828}$ $a_{39} = \frac{7}{200126180395998828} \cdot \frac{1}{3} = \frac{7}{600378541187996484}$ $a_{40} = \frac{7}{600378541187996484} \cdot \frac{1}{3} = \frac{7}{1801135623563989452}$ $a_{41} = \frac{7}{1801135623563989452} \cdot \frac{1}{3} = \frac{7}{5403406870691968356}$ $a_{42} = \frac{7}{5403406870691968356} \cdot \frac{1}{3} = \frac{7}{16210220612075905068}$ $a_{43} = \frac{7}{16210220612075905068} \cdot \frac{1}{3} = \frac{7}{48630661836227715204}$ $a_{44} = \frac{7}{48630661836227715204} \cdot \frac{1}{3} = \frac{7}{145891985508683145612}$ $a_{45} = \frac{7}{145891985508683145612} \cdot \frac{1}{3} = \frac{7}{437675956526049436836}$ $a_{46} = \frac{7}{437675956526049436836} \cdot \frac{1}{3} = \frac{7}{1313027869578148310508}$ $a_{47} = \frac{7}{1313027869578148310508} \cdot \frac{1}{3} = \frac{7}{3939083608734444931524}$ $a_{48} = \frac{7}{3939083608734444931524} \cdot \frac{1}{3} = \frac{7}{11817250826203334794572}$ $a_{49} = \frac{7}{11817250826203334794572} \cdot \frac{1}{3} = \frac{7}{35451752478610004383716}$ $a_{50} = \frac{7}{35451752478610004383716} \cdot \frac{1}{3} = \frac{7}{106355257435830013151148}$ $a_{51} = \frac{7}{106355257435830013151148} \cdot \frac{1}{3} = \frac{7}{319065772307490039453444}$ $a_{52} = \frac{7}{319065772307490039453444} \cdot \frac{1}{3} = \frac{7}{957197316922470118360332}$ $a_{53} = \frac{7}{957197316922470118360332} \cdot \frac{1}{3} = \frac{7}{2871591950767410355080996}$ $a_{54} = \frac{7}{2871591950767410355080996} \cdot \frac{1}{3} = \frac{7}{8614775852302231065242988}$ $a_{55} = \frac{7}{8614775852302231065242988} \cdot \frac{1}{3} = \frac{7}{25844327556906693195728964}$ $a_{56} = \frac{7}{25844327556906693195728964} \cdot \frac{1}{3} = \frac{7}{77532982670720079587186892}$ $a_{57} = \frac{7}{77532982670720079587186892} \cdot \frac{1}{3} = \frac{7}{232598948012160238761560676}$ $a_{58} = \frac{7}{232598948012160238761560676} \cdot \frac{1}{3} = \frac{7}{697796844036480716284682028}$ $a_{59} = \frac{7}{697796844036480716284682028} \cdot \frac{1}{3} = \frac{7}{2093390532109442148854046084}$ $a_{60} = \frac{7}{2093390532109442148854046084} \cdot \frac{1}{3} = \frac{7}{6280171596328326446562138252}$ $a_{61} = \frac{7}{6280171596328326446562138252} \cdot \frac{1}{3} = \frac{7}{18840514788984979339686414756}$ $a_{62} = \frac{7}{18840514788984979339686414756} \cdot \frac{1}{3} = \frac{7}{56521544366954938019059244268}$ $a_{63} = \frac{7}{56521544366954938019059244268} \cdot \frac{1}{3} = \frac{7}{169564633080864814057177732804}$ $a_{64} = \frac{7}{169564633080864814057177732804} \cdot \frac{1}{3} = \frac{7}{508693899242594442171533198412}$ $a_{65} = \frac{7}{508693899242594442171533198412} \cdot \frac{1}{3} = \frac{7}{1526081697727783326514599595236}$ $a_{66} = \frac{7}{1526081697727783326514599595236} \cdot \frac{1}{3} = \frac{7}{4578245093183349979543798785708}$ $a_{67} = \frac{7}{4578245093183349979543798785708} \cdot \frac{1}{3} = \frac{7}{13734735279549949938631396357124}$ $a_{68} = \frac{7}{13734735279549949938631396357124} \cdot \frac{1}{3} = \frac{7}{41204205838649849815894189071372}$ $a_{69} = \frac{7}{41204205838649849815894189071372} \cdot \frac{1}{3} = \frac{7}{123612617515949549447682567214116}$ $a_{70} = \frac{7}{123612617515949549447682567214116} \cdot \frac{1}{3} = \frac{7}{370837852547848648343047701642348}$ $a_{71} = \frac{7}{370837852547848648343047701642348} \cdot \frac{1}{3} = \frac{7}{1112513557643545945029143104927044}$ $a_{72} = \frac{7}{1112513557643545945029143104927044} \cdot \frac{1}{3} = \frac{7}{3337540672930637835087429314781132}$ $a_{73} = \frac{7}{3337540672930637835087429314781132} \cdot \frac{1}{3} = \frac{7}{10012622018791913505262287944343396}$ $a_{74} = \frac{7}{10012622018791913505262287944343396} \cdot \frac{1}{3} = \frac{7}{30037866056375740515786863833030188}$ $a_{75} = \frac{7}{30037866056375740515786863833030188} \cdot \frac{1}{3} = \frac{7}{90113598169127221547360591499090564}$ $a_{76} = \frac{7}{90113598169127221547360591499090564} \cdot \frac{1}{3} = \frac{7}{270340794507381664642081774497271692}$ $a_{77} = \frac{7}{270340794507381664642081774497271692} \cdot \frac{1}{3} = \frac{7}{811022383522144993926245323491815076}$ $a_{78} = \frac{7}{811022383522144993926245323491815076} \cdot \frac{1}{3} = \frac{7}{2433067150566434981778735970475445228}$ $a_{79} = \frac{7}{2433067150566434981778735970475445228} \cdot \frac{1}{3} = \frac{7}{7299201451699304945336207911426335684}$ $a_{80} = \frac{7}{7299201451699304945336207911426335684} \cdot \frac{1}{3} = \frac{7}{21897604355097914835908623734279007052}$ $a_{81} = \frac{7}{21897604355097914835908623734279007052} \cdot \frac{1}{3} = \frac{7}{65692813065293744507725871202837021156}$ $a_{82} = \frac{7}{65692813065293744507725871202837021156} \cdot \frac{1}{3} = \frac{7}{197078439195881233523177613608511063468}$ $a_{83} = \frac{7}{197078439195881233523177613608511063468} \cdot \frac{1}{3} = \frac{7}{591235317587643700569532840825533190404}$ $a_{84} = \frac{7}{591235317587643700569532840825533190404} \cdot \frac{1}{3} = \frac{7}{1773705952762931101708598522476599571212}$ $a_{85} = \frac{7}{1773705952762931101708598522476599571212} \cdot \frac{1}{3} = \frac{7}{5321117858288793305125795567429798713636}$ $a_{86} = \frac{7}{5321117858288793305125795567429798713636} \cdot \frac{1}{3} = \frac{7}{15963353574866379915377386702289396140808}$ $a_{87} = \frac{7}{15963353574866379915377386702289396140808} \cdot \frac{1}{3} = \frac{7}{47889060724599139746132160106868188422424}$ $a_{88} = \frac{7}{47889060724599139746132160106868188422424} \cdot \frac{1}{3} = \frac{7}{143667182173797419238396480320604565267272}$ $a_{89} = \frac{7}{143667182173797419238396480320604565267272} \cdot \frac{1}{3} = \frac{7}{430991546521392257715189440961813695801816}$ $a_{90} = \frac{7}{430991546521392257715189440961813695801816} \cdot \frac{1}{3} = \frac{7}{1292974639564176773145568322885441087405448}$ $a_{91} = \frac{7}{1292974639564176773145568322885441087405448} \cdot \frac{1}{3} = \frac{7}{3878923918692530319436704968656323262216344}$ $a_{92} = \frac{7}{3878923918692530319436704968656323262216344} \cdot \frac{1}{3} = \frac{7}{11636771756077590958309114905968969786649032}$ $a_{93} = \frac{7}{11636771756077590958309114905968969786649032} \cdot \frac{1}{3} = \frac{7}{34910315268232772874927344717906909359947096}$ $a_{94} = \frac{7}{34910315268232772874927344717906909359947096} \cdot \frac{1}{3} = \frac{7}{104730945804698318624782034153720728079841288}$ $a_{95} = \frac{7}{104730945804698318624782034153720728079841288} \cdot \frac{1}{3} = \frac{7}{314192837414094955874346102461162184239523864}$ $a_{96} = \frac{7}{314192837414094955874346102461162184239523864} \cdot \frac{1}{3} = \frac{7}{942578512242284867623038307383486552718571592}$ $a_{97} = \frac{7}{942578512242284867623038307383486552718571592} \cdot \frac{1}{3} = \frac{7}{2827735536726854602869114922150459658155714776}$ $a_{98} = \frac{7}{2827735536726854602869114922150459658155714776} \cdot \frac{1}{3} = \frac{7}{8483206608180563808607344766451378974467144328}$ $a_{99} = \frac{7}{8483206608180563808607344766451378974467144328} \cdot \frac{1}{3} = \frac{7}{25449619824541691425822034299354136923401432984$

(9)

$$a_1, a_1q, a_1q^2, a_1q^3, \dots$$

$$q \neq 1$$

$$a_1q - a_1 = a_1(q-1) \quad \times q$$

$$a_1q^2 - a_1q = a_1q(q-1) \quad \times q$$

$$a_1q^3 - a_1q^2 = a_1q^2(q-1)$$

$$a_1, a_2, a_3, \dots, a_n$$

$$a_n = a_1 + (n-1)d$$

$$s_n = a_1 + a_2 + a_3 + \dots + a_n$$

$$s_n = a_n + a_{n-1} + a_{n-2} + \dots + a_1$$

$$2s_n = (a_1 + a_n) + (a_2 + a_{n-1}) + \dots + (a_n + a_1)$$

$$a_1 + a_n$$

$$2s_n = n(a_1 + a_n) \rightarrow s_n = \frac{n(a_1 + a_n)}{2} \rightarrow \frac{n(a_1 + a_1 + (n-1)d)}{2}$$

$$s_n = \frac{n}{2}(2a_1 + (n-1)d)$$

(10)

$$a, aq, aq^2, \dots, aq^{n-1}$$

$$s_n = a + aq + aq^2 + \dots + aq^{n-1}$$

$$\times q \rightarrow qs_n = aq + aq^2 + aq^3 + \dots + aq^n$$

$$s_n - qs_n = a - aq^n \rightarrow s_n(1-q) = a(1-q^n) \rightarrow s_n = \frac{a(1-q^n)}{1-q}$$