۱۷٫۵

نسترن اسماعیلی

۱- $a_n = \frac{1}{2}, 1, 2, \ldots \Rightarrow \frac{1}{2} \times 2^{n-1} = 2^{n-2}$ (۱٫۵)

الف) $\frac{1}{2} \times 2^9 = 2^{-1} \times 2^9 = 2^8 = 256$

ب) $\frac{a_{17}}{a_{13}}$ ، $a_{17} = aq^{16} \rightarrow \frac{1}{2} \times 2^{16} = 2^{15}$

$a_{13} = aq^{12} \rightarrow \frac{1}{2} \times 2^{12}$ ⇒ $\frac{2^{15}}{2^{11}} = 2^4 = 8$ — $q^4 = 16$

ج) $a_4, a_{10} \rightarrow a_4 = aq^3 \rightarrow \frac{1}{2} \times 2^3 = 2^2 = 4$

$a_{10} = aq^9 \rightarrow \frac{1}{2} \times 2^9 = 2^8 = 256$ ، $a, b, c \rightarrow b^2 = ac$

$b^2 = 256 \times 4$

$b^2 = 1024$

د) $a_n = 128$ ، $b = 2^5 = 32$

$\frac{1}{2} \times 2^{n-1} = 128 \rightarrow \frac{1}{2} \times 2^{n-1} = 2^7 \rightarrow n-2 = 7$ ، $n = 9$

۲- $a_5 = 12$ / $a_8 = 96$ / $a_{10} = a_5 \times q^5 = 12 \times 2^5 = 12 \times 32 = 384$ (۲)

$\frac{aq^7}{aq^4} = q^3 = \frac{96}{12} \rightarrow q^3 = 8 \rightarrow q = 2$

۳- $a_1 \times a_2 \times a_3 \times a_4 \times a_5 = 243$

$a \times aq \times aq^2 \times aq^3 \times aq^4 = a^5 q^{(0+1+2+3+4)} = a^5 q^{10} = (aq^2)^5$ (۲)

$a_3^5 = 243 \rightarrow 243 = 3^5$

$a_3 = 3$ / $a_1 a_5 = (aq^2)^2 = (a_3)^2 = 3^2 = 9$

۴- $2^a, 4\sqrt{2}, 2^b \rightarrow q = \frac{a_2}{a_1} = \frac{4\sqrt{2}}{3}$ (۰)

$a_3 = a_2 q = 4\sqrt{2} \times \frac{4\sqrt{2}}{3} = \frac{16 \times 2}{3} = \frac{32}{3}$

$2^a \times 2^b = (4\sqrt{2})^2 \rightarrow 2^{a+b} = 2^5$ ، $\frac{a+b}{2} = \frac{5}{2}$

$a_n = aq^{n-1}$ (۲)

$d_n = a_{n+1} - a_n = aq^n - aq^{n-1} = aq^{n-1}(q-1)$

قدر نسبت دنباله اختلاف $= r$

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$a_3 = 1-x$ $\qquad \frac{a_7}{a_3} = q^4 \rightarrow \frac{x}{1-x}$

$a_7 = x$

$a_{11} = x+1$ $\qquad \frac{a_{11}}{a_3} = q^8 = \frac{x+1}{1-x}$

$$\left(\frac{x}{1-x}\right)^2 = \frac{x+1}{1-x} \Rightarrow x = \frac{1}{\sqrt{2}}$$

۸-

$a_n = 2, 6, 18, 54, \dots = 2 \times 3^{n-1}$

$S_{10} = 2\left(\frac{3^{10}-1}{3-1}\right) = 2 \times \frac{59048}{2} = 59048$

$P_{10} = \sqrt{(2 \times 2 \times 3^9)^{10}}$ $\qquad a_{10} = aq^9 = 2 \times 3^9$

$\sqrt{(4 \times 3^9)^{10}} = (4 \times 3^9)^5 = 4^5 \times 3^{45}$

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۳

$a \times aq \times aq^2 = a^3q^3 = 27 \Rightarrow (aq)^3 = 27 \rightarrow aq = 3$

$a_2 = 3$

$a + 3 + aq^2 = 13 \rightarrow \frac{3}{q} + 3 + 3q = 13$

$q + \frac{1}{q} = \frac{10}{3} \Rightarrow 3q^2 - 10q + 3 = 0$

$a \cdot 3 = 3 \Rightarrow a = 1 \quad , q = 3$

$1, 3, 9$

۶-

۲

$a_1 + a_4 = 28 \;/\; a_2 + a_3 = 12$

$a_1 + a_4 = a + aq^3 = a(1+q^3) = 28$

$a_2 + a_3 = aq + aq^2 = aq(1+q) = 12$

$\frac{aq(1+q)}{a(1+q^3)} = \frac{12}{28} = \frac{3}{7}$ $\qquad 7q(1+q) - 3(1+q^3) \Rightarrow$

$3q^3 - 7q^2 - 7q + 3 = 0$

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۱۵ -

الف) $a, a+d, a+2d, \dots, a+(n-1)d$

$S_n = a + (a+d) + (a+2d) + \dots (a+(n-1)d)$

$$\begin{cases} S_n = a + (a+d) + \dots + (a+(n-1)d) \\ S_n = (a+(n-1)d) + (a+(n-2)d) + \dots + a \end{cases}$$

$2S_n = n(2a+(n-1)d)$

$$\boxed{S_n = \frac{n}{2}(2a+(n-1)d)}$$

ب) $a, aq, aq^2, \dots, aq^{n-1}$

$S_n = a + aq + aq^2 + \dots + aq^n$

$qS_n = aq + aq^2 + \dots + a$

$S_n - qS_n = a - aq^n$

$(1-q)S_n = a(1-q^n)$

$S_n = a \times \left(\frac{1-q^n}{1-q}\right) = a\left(\frac{q^n-1}{q-1}\right)$