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نام و نام خانوادگی .... پاسخنامه تشریحی تکلیف شماره ..۱۴.. کلاس ....

۱	$a_n = \frac{1}{2}, 1, 2, \ldots$ $a_1 = \frac{1}{2}$ $q = 2$ $a_4 = a_1 q^3 = 4$ الف) $a_{10} = \frac{1}{2} \times 2^9 = 2^8$ ب) $\frac{a_3 q^4}{a_3} = 2^4 = 16$ ج) $b^2 = ac \to b^2 = a_4 a_{10} = 2^{10} = b^2 \to b = 2^5$ د) $128 = \frac{1}{2} \times (2)^{n-1} = 256 = 2^{n-1} \to 2^8 = 2^{n-1} \to n-1 = 8 \to n = 9$ ۳
۲	$a_5 = 12$ $a_8 = 96$ $a_5 q^3 = 96$ → $q^3 = 8$, $q = 2$ $a_{10} = a_5 q^5$ $12 \times 32 = 384$ ۳
۳	$a_1 \times a_2 \times a_3 \times a_4 \times a_5 = 243 \to (a_3)^5 = 243 \to$ الف) $= a_3 = a_3 = 3$ ب) $a_1 a_5 = (a_3)^2 \to 9$ ۳
۴	$2^a, 4\sqrt{2}, 2^b$ $32 = 2^{a+b}$ $2^5 = 2^{a+b}$ $5 = a+b$ $\frac{a+b}{2} = 5/2$ واسطه حسابی ۳
۵	$a_3 = 1-x$ $x^2 = 1-x^2$ $a_7 = x$ $2x^2 = 1$ $a_{11} = x+1$ $x^2 = 1/2$ $x = \pm\sqrt{\frac{1}{2}}$ $x = 1$ $x = \pm\frac{1}{\sqrt{2}} \to -\frac{\sqrt{2}}{2}$ مورد قبول نیست ۱٫۷۵

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$a_1 + a_4 = 48$
$a_2 + a_3 = 12$
$a_1 + a_1q^3 = 48$
$a_1q + a_1q^2 = 12$
$a_1(1+q^3) = 48$
$a_1q(1+q) = 12$

$\frac{(1+q)(1-q+q^2)}{q(1+q)} = \frac{48}{12}$ → $7q = 3 - 3q + 3q^2$

$3q^2 - 10q + 3 = 0$

$\Delta = 100 - 36 = 64$

$q = \frac{10 \pm 8}{6} = \frac{1}{3}, 3$

$a_1 = 27$

$a_2 = 9$

$a_3 = 3$

$a_4 = 1$

$a_5 = \frac{1}{3}$

$a_6 = \frac{1}{9}$

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$a_1 + a_2 + a_3 = 13$ → $a_1 + a_1q + a_1q^2 = 13$ → $a_1(1+q+q^2) = 13$

$a_1 a_2 a_3 = 27$ → $a_1 \cdot a_1q \cdot a_1q^2 = 27$ → $(a_1q)^3 = 27$ → $a_1q = 3 = a_2$

$a_1 + \frac{3}{a_1} = 10$ → $\frac{a_1^2 + 3}{a_1} = 10$ → $10a_1 = 3 + a_1^2$

$3 + a_1^2 - 10a_1 = 0$

$(a_1 - 9)(a_1 - 1) = 0$

$a_1 = 9$ $a_1 = 1$

$q = \frac{3}{a_1}$

$q = \frac{1}{3}$ یا 3

$a_1 = 1$ یا 9

$a_2 = 3$ یا 3

$a_3 = 9$ یا 1

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$a_n = 2, 6, 18, \dots$ $q = 3$

الف) $2\left(\frac{3^{10}-1}{2}\right) = 3^{10} - 1 = 59049$

$a_4 = 2 \times 3^3$

$a_5 = 2 \times 3^4$

ب) $a_1\ a_2\ a_3\ a_4\ a_5\ a_6\ a_7\ a_8\ a_9\ a_{10} = (a_4 a_5)^5$

$(2 \times 3^3 \times 2 \times 3^4)^5 = (4 \times 3^7)^5 = 4^5 \times 3^{35}$

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$a_1, a_1q, a_1q^2, a_1q^3, \dots$

$a_1q - a_1 = a_1(q-1) = b_1$) $\times q$

$a_1q^2 - a_1q = a_1q(q-1) = b_2$

$a_1q^3 - a_1q^2 = a_1q^2(q-1) = b_3$) $\times q$

$b_n = a_1(q-1)q^{n-1}$

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الف) $S_n = a + (a+d) + (a+2d) + \dots + (a+(n-1)d)$ → $S_n = (a+(n-1)d) + \dots + a$

$S_n = (a+(n-1)d) + (a+(n-2)d) + \dots a$ → $2S_n = n(2a + (n-1)d)$ → $S_n = \frac{n}{2}(2a + (n-1)d)$

ب) $a + aq + aq^2 + \dots aq^{n-1} = S_n$

$qS_n = aq + aq^2 + \dots + aq^n$ → $S_n - qS_n = a - aq^n$ → $S_n(1-q) = a(1-q^n)$

$S_n = a\left(\frac{1-q^n}{1-q}\right) = a\left(\frac{q^n - 1}{q-1}\right)$