آرام صنوبریان دهم A. تکلیف ۱۳

(۱) $a_n = \frac{1}{2}(2)^{n-1}$

الف) $a_{10} = \frac{1}{2}(2)^9 = 256$

ب) $\frac{a_1 q^{16}}{a_1 q^{12}} = q^4 = 2^4 = 16$

ج) $b^2 = ac \to \begin{cases} a = 2^2 \\ c = 2^8 \end{cases} \Rightarrow 2^{10} = b^2$

$b = 2^5 = 32$

(۲) $\frac{a_8}{a_5} = \frac{aq^7}{aq^4} = q^3 = \frac{96}{12} = 8 \to q = 2$

$a_5 = a_1 q^4 = a_1 \times 2^4 = 12 \to a_1 = \frac{3}{4}$

$a_n = \frac{3}{4}(2)^{n-1}$

$a_{10} = \frac{3}{4}(2)^9 = 384$

(۳) $a_1 \times a_2 \times a_3 \times a_4 \times a_5 = 3^5$

$a_n = 3, 3, 3, \dots \quad q = 1$

الف) 3

ب) $3^2 = 9$

(۴) $2^a, 4\sqrt{2}, 2^b$

$b^2 = ac \to 2^a . 2^b = 32 \to 2^{a+b} = 2^5$

$a + b = 5$

$b = \frac{a+c}{2} = \frac{a+b}{2} = \frac{5}{2} = 2/5$

(۵) $x+1, x, 1-x$

$b^2 = ac \to x^2 = x - x^2 + 1 - x \Rightarrow 2x^2 = 1$

$x = \pm\sqrt{\frac{1}{2}}$

(۶) $a_1 + a_4 = 28 \quad , \quad a_2 + a_3 = 12$

$$\frac{a_1(1+q^3)}{q a_1(1+q)} = \frac{1+q^3}{q(1+q)} \to \frac{(1+q)(1-q+q^2)}{q(1+q)} = \frac{7}{3}$$

$3(1-q+q^2) = 7q \Rightarrow 3q^2 - 10q + 3 = 0$

$\Delta = 100 - 36 = 64 \to \frac{10 \pm \sqrt{64}}{6} = 3 , \frac{1}{3}$

$a_1(1+q^3) = 28$

$q = 3 \to (a_1 + 27a_1) = a_1(1+27) = 28 \to \boxed{a_1 = 1}$

$1, 3, 9, 27$ ← بزرگترین

(۷) $a_1 + a_2 + a_3 = a_1(1+q+q^2) = 13$

$a_1 . a_2 . a_3 = a_1^3 . q^3 = 27 \to a_1 q = 3$

$q = \frac{3}{a_1}$

$a_1 + \underbrace{a_1 q}_{3} + a_1 q^2 = 13$

$a_1(1+q^2) = 10 \to a_1 + \frac{9a_1}{a_1^2} = \frac{a_1^2 + 9}{a_1} = 10$

$a_1^2 - 10a_1 + 9 = 0$

$a_1 = 1 , 9 \qquad q = 3 , \frac{1}{3} \qquad a_n = 1, 3, 9$

$9, 3, 1$

(۸) $2, 6, 18, \dots \quad a_n = 2(3)^{n-1}$

الف) $S_{10} = a_1\left(\frac{q^n - 1}{q - 1}\right) = \cancel{2}\left(\frac{3^{10} - 1}{\cancel{3-1}}\right) = 3^{10} - 1$

ب) $P_n = \sqrt{(a_1 . a_n)^n} \to \sqrt{(2 \times 2 \times 3^9)^{10}} = (4 \times 3^9)^5$

$$a \,,\, aq \,,\, aq^2 \,,\, \dots$$ (9)

دنباله جدید $= \underbrace{aq - a}_{a(q-1)} \,,\, \underbrace{aq^2 - aq}_{aq(q-1)} \,,\, \underbrace{aq^3 - aq^2}_{aq^2(q-1)} \,,\, \dots$

$$\longrightarrow a_n = a_1 (q)^{n-1} \Rightarrow a(q-1) \times q^{n-1}$$

قدر نسبت $= q$

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(10)

الف) $S_n = \frac{n}{2}\left(2a + (n-1)d\right) = \frac{n}{2}(a_1 + a_n)$

$$S_n = a_1 + a_2 + a_3 + \dots + a_n \Rightarrow S_n = a_1 + (a_1 + d) + (a_1 + 2d) + (a_1 + 3d) + \dots + a_n$$

$$S_n = a_n + a_{n-1} + a_{n-2} + \dots + a_1 \Rightarrow S_n = a_n + (a_n - d) + \dots + a_1$$ ← حالا از آخر به اول

جمع

$$2S_n = \underbrace{(a_1 + a_n) + (a_1 + a_n) + \dots + (a_1 + a_n)}_{n \text{ تا}} \Rightarrow 2S_n = n(a_1 + a_n)$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

ب) $S_n = a_1 \left(\frac{1 - q^n}{1 - q}\right)$

$$S_n = a_1 + a_1 q + a_1 q^2 + \dots + a_1 q^{n-1} \xrightarrow{\times q} qS_n = a_1 q + a_1 q^2 + \dots + a_1 q^n$$

کم می‌کنیم

$$S_n - qS_n = a_1 - a_1 q^n$$

$$S_n(1 - q) = a_1(1 - q^n)$$

$$S_n = a_1 \frac{(1 - q^n)}{(1 - q)}$$