

آیا شعری - کرده هم قدر - تلف سوار ۱۴

1- الف) $\rightarrow a_n, a_2^{n-1} \rightarrow \frac{r^{-1}}{r} \times r^{n-1} = r^{n-2}, a_n \Rightarrow a_1, r^n, 259$ (115)

ب) $\frac{a_{11}}{a_{12}} = \frac{a_1}{a_2} = r^k, (14)$

ج) $q^{n+1} = \frac{b}{a} \rightarrow q^r = \frac{a_{11}}{a_2} = \frac{a_1}{a_2} = r^9 \rightarrow q = r^9, 1 \rightarrow r^9, 32, 254$ ✓

د) $128, r^9 = r^{n-2} \rightarrow n-2=9 \rightarrow n=11$ (جدول ۱۱ ام)

$128 = \frac{1}{r} \times r^{n-1}$
 $r^{n-1} = 252, n-1=11, n=12$

2- $a_5=12, a_8=99 \rightarrow \frac{a_n}{a_m} = q^r, \frac{99}{12}, 1=2^3 \rightarrow q=2 \rightarrow a_1, a_n \times q^r = 99 \times 4, 396$ (117)

3- الف) $a_1 \times a_2 \times a_3 \times a_4 \times a_5, a^5 \cdot d^{10}, (ad^2)^5, 243, 3^5 \rightarrow ad^2, a_5=3$

ب) $a_1 \times a_5, a^5 \cdot d^4, (ad^2)^5, 3^5=9$ (9)

4- $(\sqrt{2})^r = 2^a \times 2^b \rightarrow 2^{r/2} = 2^{a+b} \rightarrow a+b=5 \Rightarrow a=1, b=3 \rightarrow 2, 8$

$\Rightarrow \frac{r}{2} = a+b = 4 \Rightarrow r=8$

5- جمله هفت، واسطه هندسی بین جمله سوم و یازدهم است: $\frac{a_{11}}{a_7} = \frac{a_7}{a_3} = q^2 \Rightarrow (a_7)^2 = a_{11} \cdot a_3$

$\Rightarrow q^2 = \frac{(1+x)(1-x)}{(1-x)^2} = \frac{1-x^2}{(1-x)^2} \rightarrow 1-x^2 = (1-x)^2 \rightarrow x = \pm \sqrt{\frac{1}{2}}$ (فقط ۱/۲ درست) (117)

$a+aq^r, 21 \rightarrow a(1+q^r) = a(1+q)(1-q+q^r) = 21 \rightarrow \frac{a_1+a_2}{a_3+a_4} = \frac{a(1+q)(1-q+q^r)}{aq(1+q)} = \frac{21}{12} = \frac{7}{4} \rightarrow 3-3q+q^2 = \frac{7}{4}$

$\Rightarrow 3-3q+q^2 = \frac{7}{4} \rightarrow 4-12q+4q^2 = 7 \rightarrow \Delta = b^2-4ac, 100-4(4)(-3), 48 \rightarrow q = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{10 \pm \sqrt{136}}{8} = \frac{10 \pm 2\sqrt{34}}{8} = \frac{5 \pm \sqrt{34}}{4}$

$\Rightarrow q_1 = \frac{11}{4} = 2.75, q_2 = \frac{1}{4} = 0.25$ (بزرگترین قدر نسبت) $\Rightarrow a=1 \rightarrow aq^3 = 1 \times 27 = 27$ (27)

$\frac{a}{2} \times a \times aq = 27 \rightarrow a^3 = 27 \rightarrow a=3$

$\frac{a}{2} + a + aq = 13 \rightarrow \frac{3}{2} + 3 + 3q = 13 \rightarrow 3q = 6.5 \rightarrow q = \frac{13}{6}$

$\frac{a}{2} = 3 \rightarrow \frac{3}{2}, 3, 3 \times \frac{13}{6} \rightarrow 1.5, 3, 6.5$

$\frac{a}{2} = \frac{1}{3} \rightarrow \frac{1}{6}, \frac{1}{3}, \frac{1}{3} \times \frac{13}{6} \rightarrow 0.16, 0.33, 0.72$

1- الف) $S_n = a \left(\frac{q^n - 1}{q - 1} \right) \Rightarrow S_1 = 2 \left(\frac{2^1 - 1}{2 - 1} \right) = 2^1 - 1 = 1, 59.48 \checkmark$

ب) $a_0 = 18 \times 9, 192 \rightarrow P_n = (18 \times 9)^n \rightarrow P_1 = 162$

ج) $a_n \rightarrow a, aq, aq^2, \dots, aq^{n-1}$
 $b_n \rightarrow a(q-1), aq(q-1), aq^2(q-1), \dots, aq^{n-1}(q-1)$
 $\Rightarrow \frac{b_n}{b_1} = \frac{aq^{n-1}(q-1)}{a(q-1)} = q^{n-1} \Rightarrow \frac{b_n}{b_1} = \frac{aq^{n-1}(q-1)}{a(q-1)} = q^{n-1} = q^{n-1-n+1} = q^1 = q \checkmark$

د) $S = \frac{n(a_1 + a_n)}{2} = \frac{n}{2} (a + a_n)$, $a_n = a + (n-1)d \Rightarrow S = \frac{n}{2} (a + a + (n-1)d) = \frac{n}{2} (2a + (n-1)d)$

هـ) $S_n = a + aq + aq^2 + \dots + aq^{n-1} \xrightarrow{\times q} qS_n = aq + aq^2 + aq^3 + \dots + aq^n$

$\Rightarrow qS_n - S_n = (aq + aq^2 + aq^3 + \dots + aq^n) - (a + aq + aq^2 + \dots + aq^{n-1}) = aq^n - a = a(q^n - 1)$

$\Rightarrow S_n(q-1) = a(q^n - 1) \rightarrow S_n = a \left(\frac{q^n - 1}{q - 1} \right)$