

$$a_1 = \frac{1}{2}, q = 2 \rightarrow a_n = \frac{1}{2} (2)^{n-1}$$

①

الف) $a_{10} = \frac{1}{2} (2)^9 = 256$

ب) $a_{14} = a q^{13}, a_{13} = a q^{12} \rightarrow \frac{a q^{14}}{a q^{13}} = q = 2^4 = 16$

ج) $a_4 = \frac{1}{8} 2^{3+2} = 4, a_{10} = 256 \rightarrow b^2 = \frac{256 \times 4}{10 \times 2^4} \rightarrow b = \sqrt{10 \times 2^4} = 32$

د) $\frac{1}{2} (2)^{n-1} = 128 \rightarrow 2^{n-1} = 256 = 2^8 \rightarrow n-1 = 8 \rightarrow n = 9$

$$a_5 = a q^4 = 12, a_n = a q^9 q^4 \rightarrow \frac{a q^9}{a q^4} = \frac{96}{12} \rightarrow q^5 = 8 \rightarrow q = 2$$

②

$$a 2^4 = 12 \rightarrow a = \frac{3}{4} \quad ; \quad a_{10} = \frac{3}{4} \times 2^9 \rightarrow 312$$

الف) $a_3 = 2^4 2^3 = 3^7 \rightarrow a_3 = 3$

③

ب) $a_1 \times a_5 = a_3^2 = 3^2 = 9$

$$(4\sqrt{2})^2 = 2^a \times 2^b \rightarrow 32 = 2^5 = 2^{a+b} \rightarrow a+b = 5 \rightarrow c = \frac{a+b}{2}$$

④

$$c = \frac{5}{2} = 2.5$$

$$a q^2 = 1-x, a q^4 = x, a q^{10} = x+1$$

⑤

$$\frac{a q^{10}}{a q^4} = \frac{a q^4}{a q^2} = q^6 \quad ; \quad \frac{x+1}{x} \times \frac{x}{1-x} \rightarrow x^2 = 1-x^2 \rightarrow 2x^2 = 1 \rightarrow x = \pm \frac{1}{\sqrt{2}} = \pm \frac{\sqrt{2}}{2}$$

$$x = \frac{\sqrt{2}}{2}$$

$$a + ar^3 = 21 = a(1+r^3), ar + ar^2 = 12 \Rightarrow ar(1+r)$$

⑥

$$\frac{a(1+r^3)}{ar(1+r)} = \frac{21}{12} = \frac{7}{4} \rightarrow \frac{1+r^3}{r(1+r)} = \frac{7}{4} \rightarrow 4(1+r^3) = 7r(1+r) \rightarrow 4 + 4r^3 = 7r + 7r^2 \rightarrow 4r^3 - 7r^2 - 7r + 4 = 0$$

$$\Delta = (-7)^2 - 4 \times 4 \times 4 = 49 - 64 = -15 \rightarrow \sqrt{\Delta} = \sqrt{15}$$

$$r = \frac{10 \pm \sqrt{15}}{4} \rightarrow r = 3 \text{ or } r = \frac{1}{3}$$

مثال: 1, 3, 9, 27

$$a = \frac{12}{r(1+r)} = \frac{12}{3 \times 4} = 1 \rightarrow r = 3$$

بزرگترین جمله: 27

$$a + ar + ar^2 = 13$$

⑦

$$a \times ar \times ar^2 = a^3 r^3 = 27 \rightarrow ar = \sqrt[3]{27} = 3 \rightarrow a = \frac{3}{r}$$

$$\frac{3}{r}(1+r+r^2) = 13 \rightarrow 3 + 3r + 3r^2 = 13r \rightarrow 3r^2 - 10r + 3 = 0 \rightarrow r = 3 \text{ or } r = \frac{1}{3}$$

$$r = 3 \rightarrow a = \frac{3}{3} = 1 \rightarrow \text{جمله: } 1, 3, 9$$

$$r = \frac{1}{3} \rightarrow a = 1 \rightarrow \text{جمله: } 9, 3, 1$$

$$a_1 = 2, q = 3$$

⑧

$$\text{الف) } S_{10} = X \left(\frac{3^{10} - 1}{3 - 1} \right) = 3^{10} - 1 = 59048$$

$$\text{ب) } (a_1 \times a_{10})^{\frac{10}{2}} = 5 \rightarrow a_1 = 2, a_{10} = 2 \times 3^9 = 392664 \left\{ \begin{array}{l} (2 \times 2 \times 3^9)^5 = 2^{10} \times 3^{45} \end{array} \right.$$

$$a_n = a, aq, aq^2, aq^3 \rightarrow (aq - a), (aq^2 - aq), (aq^3 - aq^2) \quad (9)$$

$$a(q-1), a(q-1)q, a(q-1)q^2, \dots \rightarrow \text{دنباله هندسی}$$

$$a_1 = a(q-1), d = q$$

(10)

الف)

$$a_n = a_1, a+d, a+2d, \dots, a+(n-1)d$$

$$S_n = a + (a+d) + (a+2d) + \dots + [a+(n-1)d]$$

$$S_n = [a+(n-1)d] + [a+(n-2)d] + \dots + (a+d) + a$$

$$\left\{ \begin{array}{l} \rightarrow a + [a+(n-1)d] = 2a + (n-1)d \\ \rightarrow (a+d) + [a+(n-2)d] = 2a + (n-1)d \end{array} \right\} \begin{array}{l} \text{هر جفت } 2a + (n-1)d \\ \leftarrow n \text{ جفت داریم} \end{array}$$

$$\Rightarrow 2S_n = n(2a + (n-1)d) \rightarrow S_n = \frac{n}{2}(2a + (n-1)d)$$

$$\text{ب) } \left(S_n = a + aq + aq^2 + \dots + aq^{n-1} \right) \xrightarrow{\times q} \left. \begin{array}{l} - qS_n = aq + aq^2 + aq^3 + \dots + aq^n \end{array} \right\} \text{از هم کم می کنیم}$$

$$\rightarrow qS_n - S_n = aq^n - a \rightarrow S_n(q-1) = a(q^n - 1)$$

$$\rightarrow S_n = a \left(\frac{q^n - 1}{q - 1} \right)$$