

سوال ١١

$$a_1 = \frac{1}{r}, q = r$$

الف) $a_{10} = \frac{1}{r} \times r^9 = 254$

ب) $\frac{a_{14}}{a_{13}} = q = r = 12$

ج) $b^r = ac \Rightarrow \sqrt{r \times 254} = \sqrt{r^{10}} = r^5 = 32$

د) $a_n = a_1 q^{n-1} \Rightarrow 12 \times 1 = \frac{1}{r} \times r^{n-1} \Rightarrow 254 = r$
 $\Rightarrow n-1 = 10 \Rightarrow n = 11$

سوال ١٢

$$q^{n-m} = \frac{a_n}{a_m} \Rightarrow q^{10-5} = \frac{94}{12} \Rightarrow q^5 = 1 \Rightarrow q = 1$$

$$a_n = a_m \times q^{n-m} \Rightarrow a_{10} = a_5 \times r^{10-5} \Rightarrow a_{10} = 94 \times 2 = 188$$

سوال ١٣

$$\frac{a_8}{q^8} \times \frac{a_6}{q^6} \times a_4 \times (a_4 \times q^4) \times (a_6 \times q^6) = a_8^2 = 256$$

الف) $\Rightarrow a_8 = 16$

ب) $\frac{r}{q^2} \times (16 \times q^2) = 9$

سوال ١٤

$$\sqrt{a} = r^{\frac{a}{r}}, b^r = ac$$

$$\Rightarrow (r^{\frac{a}{r}})^r = r^a \times r^b \Rightarrow r^a = r^a \times r^b \Rightarrow a+b=a$$

$$\frac{a+b}{r} = \frac{a}{r}$$

سوال ١٥

$$11 - v = 2, v - c = 4$$

$$\Rightarrow a^r = (1-a)(n+1) \Rightarrow n^r = 1 - n^r \Rightarrow n = \pm \sqrt{\frac{1}{r}}$$

$$\Rightarrow \frac{1}{\sqrt{r}} \times \frac{\sqrt{r}}{\sqrt{r}} = \pm \frac{\sqrt{r}}{r}$$

سوال ١٦

$$a_1 + aq^9 = 21 \Rightarrow a(1-q^9) = 21$$

$$aq + aq^9 = 12 \Rightarrow aq(1+q^8) = 12$$

$$\frac{a_1(1+q^9)}{aq(1+q)} = \frac{21}{12} \Rightarrow \frac{(1+q^9)(1-q+q^2)}{q(1+q)} = \frac{7}{4}$$

$$\Rightarrow r(1-q+q^2) = 7q \Rightarrow 2r - rq - 7q = 7q \Rightarrow 2rq^2 - 10q + 2 = 0 \Rightarrow q = \frac{-10 \pm \sqrt{100-16}}{4}$$

$$\Rightarrow q = 2 \rightarrow 2a_1(1+2) = 12 \Rightarrow a_1 = 2$$

$$\Rightarrow q = \frac{1}{2} \rightarrow a_1 q(1+q) = 12 \Rightarrow a_1 = 24$$

$$q = 2 \Rightarrow 1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024$$

سوال ١٧

$$\frac{a_r}{q} \times a_r \times aq = 27 \Rightarrow a_r^2 = 27 \Rightarrow a_r = 3$$

$$\Rightarrow a_1 + a_r + a_c = 10 \Rightarrow a_1 + a_c = 10$$

$$\Rightarrow a_1(1+q^r) = 10 \Rightarrow (\frac{3}{q})(1+q^r) = 10$$

$$\Rightarrow (\frac{3}{q})(\frac{3}{q} \times 1) + (\frac{3}{q} \times q^r) = 10$$

$$\Rightarrow \frac{9}{q} + 3q = 10 \Rightarrow 9 + 3q^2 = 10q$$

$$\Rightarrow 3q^2 - 10q + 9 = 0 \Rightarrow q = \frac{10 \pm 1}{6} \Rightarrow \frac{1}{2}$$

$$q = 2 \rightarrow 1, 2, 4$$

$$q = \frac{1}{2} \rightarrow 9, 4.5, 2.25$$

سوال ١٨

$$S_1 = r \left(\frac{r^{10}-1}{r-1} \right) = 29.81$$

سوال ١٩

$$P_n = (a_1 \cdot a_n)^n \text{ و } a_n = a_1 \cdot q^{n-1} \Rightarrow P_n = a_1^n \times q^{\frac{n(n-1)}{2}}$$

$$\Rightarrow 2^{10} \times 2^{\frac{10(10-1)}{2}} = 2^{10} \times 2^{45} = 1024 \times 2^{45}$$

سوال ٢٠

$$A = a, aq, aq^2, \dots, aq^{n-1}$$

$$B = b_1, b_2, b_3, \dots, b_n$$

$$b_1 = a_2 - a_1 = aq - a$$

$$b_2 = a_3 - a_2 = aq^2 - aq$$

$$\frac{b_2}{b_1} = \frac{aq^2 - aq}{aq - a} = \frac{aq(q-1)}{a(q-1)} = q$$

$$\frac{b_n}{b_1} = q$$

سوال ٢١

$$S_n = a_1 + (a_1 + d) + (a_1 + 2d) + \dots + (a_1 + (n-1)d)$$

$$S_n = a_n + (a_n - d) + (a_n - 2d) + \dots + (a_1 + d) + a_1$$

$$\Rightarrow S_n = (a_1 + (n-1)d) + (a_1 + (n-2)d) + \dots + a_1$$

سوال ٢٢

$$rS_n = (ra_1 + (n-1)d) + (ra_1 + (n-1)d) + \dots$$

$$rS_n = n(ra_1 + (n-1)d)$$

$$\Rightarrow S_n = \frac{n}{r}(ra_1 + (n-1)d)$$

سوال ٢٣

الف) $S_n = a_1 + aq + aq^2 + \dots + aq^{n-1}$

ب) $qS_n = aq + aq^2 + aq^3 + \dots + aq^n$

ج) $qS_n - S_n = aq^n - a$

$$\Rightarrow S_n(q-1) = a(q^n - 1) \Rightarrow S_n = a \frac{q^n - 1}{q - 1}$$

نتیجه نهایی