سوال ①

$a_1 = \frac{1}{2}$, $q = 2$

الف) $a_{10} = \frac{1}{2} \times 2^9 = 256$

ب) $\frac{a_{17}}{a_{13}} = q^{17-13} = q^4 = 2^4 = 16$

ج) $b^2 = ac \Rightarrow \sqrt{4 \times 256} = \sqrt{2^{10}} = 2^5 = 32$

د) $a_n = a_1 q^{n-1} \Rightarrow 128 = \frac{1}{2} \times 2^{n-1} \Rightarrow 256 = 2^{n-1}$

$\Rightarrow n-1 = 8 \Rightarrow n = 9$

سوال ②

الف) $S_{10} = 2\left(\frac{2^{10}-1}{2-1}\right) = 2046$

ب) $P_n = (a_1 \cdot a_n)^n$ و a_n جمله عمومی $\Rightarrow P_n = (2 \times 2^n)^n$

$\Rightarrow 2^{10} \times 2^{\frac{10(10-1)}{2}} = 2^{10} \times 2^{45} = 2^{55}$

سوال ③

$q^{n-m} = \frac{a_n}{a_m} \Rightarrow q^{8-3} = \frac{96}{12} \Rightarrow q^5 = 8 \Rightarrow q = 2$

$a_n = a_m \times q^{n-m} \Rightarrow a_{10} = a_8 \times 2^{10-8} \Rightarrow a_{10} = 96 \times 4 = 384$

سوال ④

$\frac{a_3}{q^2} \times \frac{a_5}{q} \times a_3 \times (a_3 \times q) \times (a_5 \times q^2) = a_3^5 = 243$

$\Rightarrow a_3 = 3$

ب) $\frac{3}{q^2} \times (3q^2) = 9$

سوال ⑤

$A = a_1, a_1 q, a_1 q^2, \dots, a_1 q^{n-1}$

$B \Rightarrow b_1 = a_2 - a_1 = aq - a$

$b_2 = a_3 - a_2 = aq^2 - aq$

$\Rightarrow b_n = a_{n-1} - a_n = aq^{n-1} - aq^n$

$\frac{b_2}{b_1} = \frac{b_3}{b_2} \Rightarrow \frac{aq(q-1)}{a(q-1)} = q$ $\frac{b_3}{b_2} = q$

نسبت مشترک جمله‌های B $= q$ ✓

سوال ⑥

$\sqrt{2^5} = 2^{\frac{5}{2}}$, $b^2 = ac$

$\Rightarrow (2^{\frac{5}{2}})^2 = 2^a \times 2^b \Rightarrow 2^5 = 2^{a+b} \Rightarrow a+b = 5$

$\frac{a+b}{2} = \frac{5}{2}$

سوال ⑦

$11 - 7 = 4$ $\quad 7 - 3 = 4$

$\Rightarrow x^2 = (1-x)(x+1) \Rightarrow x^2 = 1 - x^2 \Rightarrow x = \pm\frac{1}{\sqrt{2}}$

$\Rightarrow \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\pm\sqrt{2}}{2}$ فقط $\frac{\sqrt{2}}{2}$ درست!

سوال ⑧

$a_1 + aq^3 = 28 \Rightarrow a(1-q^3) = 28$

$aq + aq^2 = 12 \Rightarrow aq(1+q) = 12$

$\frac{a_1(1+q^3)}{a_1 q(1+q)} = \frac{28}{12} \Rightarrow \frac{(1+q)(1-q+q^2)}{q(1+q)} = \frac{7}{3}$

$\Rightarrow 3(1-q+q^2) = 7q \Rightarrow 3 - 3q + 3q^2 = 7q$

$\Rightarrow 3q^2 - 10q + 3 = 0 \Rightarrow q = \frac{10 \pm \sqrt{64}}{6}$

$q = 3 \rightarrow 3a_1(1+3) = 12 \Rightarrow a_1 = 1$

$q = \frac{1}{3} \rightarrow a_1 q(1+q) = 12 \Rightarrow a_1 = 27$

$q = 3 \Rightarrow 1, 3, 9$ $\quad q = \frac{1}{3} \Rightarrow 27, 9, 3, 1$

سوال ⑨

$\frac{a_2}{q} \times a_2 \times aq = 27 \Rightarrow a_2^3 = 27 \Rightarrow a_2 = 3$

$a_1 = \frac{3}{q}$

$\Rightarrow a_1 + a_2 + a_3 = 13 \xrightarrow{a_2 = 3} a_1 + a_3 = 10$

$\Rightarrow a_1(1+q^2) = 10 \Rightarrow \left(\frac{3}{q}\right)(1+q^2) = 10$

$\Rightarrow \left(\frac{3}{q}\right)\left(\left(\frac{3}{q} \times 1\right) + \left(\frac{3}{q} \times q^2\right)\right) = 10$

$\Rightarrow \frac{3}{q} + 3q = 10 \xrightarrow{\times q} 3 + 3q^2 = 10q$

$\Rightarrow 3q^2 - 10q + 3 = 0 \Rightarrow q = \frac{10 \pm 8}{6}$ $\quad 3, \frac{1}{3}$

$q = 3 \rightarrow 1, 3, 9$

$q = \frac{1}{3} \rightarrow 9, 3, 1$

سوال ⑩

الف) $S_n = a_1 + (a_1+d) + (a_1+2d) + \dots + (a_1+(n-1)d)$ ①

(برعکس نوشتن) $S_n = a_n + (a_n - d) + (a_n - 2d) + \dots + (a_1 + d) + a_1$ ②

$\Rightarrow S_n = (a_1 + (n-1)d) + (a_1 + (n-2)d) + \dots + a_1$

① + ② $\rightarrow 2S_n = \underbrace{(2a_1 + (n-1)d)}_{n \text{ جمله}} + \underbrace{(2a_1 + (n-1)d)}_{n \text{ جمله}} + \dots$

$\Rightarrow 2S_n = n(2a_1 + (n-1)d)$ ← n تا جمله داریم.

$\Rightarrow S_n = \frac{n}{2}(2a_1 + (n-1)d)$

ب) $S_n = a_1 + a_1 q + aq^2 + \dots + aq^{n-1}$ ①

$\xrightarrow{\times q}$ $qS_n = aq + aq^2 + aq^3 + \dots + aq^n$ ②

② − ① $\rightarrow qS_n - S_n = aq^n - a$

$\Rightarrow S_n(q-1) = a(q^n - 1) \Rightarrow S_n = a\frac{q^n - 1}{q-1}$

هفته نشد ۱۰

مدرسی عزیزم - موفق و تندرست باشی ❤