

آینا فکری

الف) $a_{10} = aq^9 = \frac{1}{r} \times 2^9 = 128$

$a_n = \frac{1}{r}, 1, 2, \dots$

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ب) $\frac{a_{14}}{a_{12}} = \frac{aq^{14}}{aq^{12}} = q^2 = 2^2 = 4$

ج) $q^{n+1} = \frac{b}{a} = \frac{aq^9}{aq^{12}} = q^{-3} \rightarrow \begin{matrix} n+1=4 \\ n=3 \end{matrix}$

د) $\frac{1}{r} q^{n-1} = 128 \quad q^{n-1} = 128 = 2^7 \quad \begin{matrix} n-1=7 \\ n=8 \end{matrix}$

$a_5 = 12 = aq^4, a_1 = 94 = aq^0 \rightarrow q^4 = \frac{94}{12} = 1 \rightarrow q = 2$

$a \times 2^4 = 12 \rightarrow a = 0.75 = \frac{3}{4}$

$a_{10} = \frac{3}{4} \times 2^9 = \frac{3 \times 512}{4}$

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ضرب جمله ها: $(a_1 \times a_2 \times \dots \times a_n) = (aq^0 \times aq^1 \times \dots \times aq^{n-1}) = a^n \times q^{0+1+\dots+(n-1)} = a^n \times q^{\frac{n(n-1)}{2}}$

الف) $a_3 = \sqrt{9}$

ب) $a_1 \times a_5 = a_3 \times a_3 = 3 \times 3 = 9$

Series $\leftarrow 2^a, 4\sqrt{2}, 2^b$

$(4\sqrt{2})^2 = 2^a \times 2^b$

$16 \times 2 = 2^{a+b} = 32 = 2^5 \rightarrow a+b=5$

Series $\leftarrow \frac{b+a}{n+1} = \frac{5}{1+1} = \frac{5}{2} = \frac{5}{2} \times 2 = 5$

4

5

$aq^2, aq^4, aq^{10} \rightarrow x+1, x, 1-x$

$a_3 \times a_{11} = a_7 \times a_7 \rightarrow (a_7)^2 = a_3 \times a_{11}$

$\rightarrow x^2 = (1-x)(x+1) = 1-x^2$

$2x^2 = 1$

$x^2 = \frac{1}{2} \quad x = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$$a_1 + a_2 = 18$$

$$a_2 + a_3 = 12$$

$$a + aq^3 = a(1 + q^3) = \cancel{a}(1+q)(1+q+q^2)$$

$$aq + aq^2 = \cancel{aq}(1+q) = 12$$

(4)

$$\frac{18}{12} = \frac{(1-q+q^2)}{q} = \frac{3}{q}$$

$$3q = 3(1-q+q^2)$$

$$3q = 3 - 3q + 3q^2$$

$$3q^2 - 6q + 3 = 0$$

$$q = \frac{10 \pm \sqrt{100 - 36}}{6} = \frac{10 \pm 8}{6}$$

$$\frac{18}{6} = 3$$

$$\frac{2}{6} = \frac{1}{3}$$

$$a(1 + \frac{3}{18}) = 18 \rightarrow a = 1$$

$$a(1 + (\frac{1}{3})^3) = 18$$

$$a \times \frac{28}{27} = 18$$

$$\rightarrow a = 27 \rightarrow 27, 9, 3, 1, \dots$$

بزرگترین جمله 27 است ✓

$$a_1 + a_2 + a_3 = 13 \quad \frac{a}{q} + a + aq = 13 \rightarrow \frac{3}{q} + 3 + 3q$$

$$a_1 \times a_2 \times a_3 = 27 \quad \frac{a}{q} \times a \times aq = a^3 = 27 \rightarrow a = 3$$

(5)

$$\rightarrow (\frac{3}{q} + 3 + 3q = 13) \times q \rightarrow 3 + 3q + 3q^2 = 13q \rightarrow 3q^2 - 10q + 3 = 0$$

$$q = \frac{10 \pm \sqrt{100 - 36}}{6}$$

$$\frac{18}{6} = 3$$

$$\frac{2}{6} = \frac{1}{3}$$

$$\Rightarrow 1, 3, 9, 27, \dots$$

$$a_n = 2, 4, 8, \dots$$

(6)

$$S_{10} = a \times \left(\frac{q^{10} - 1}{q - 1} \right) = 1 \times \frac{3^{10} - 1}{3 - 1}$$

$$\rightarrow q = 3, a = 1$$

$$3^{10} - 1 = (3^5 + 1)(3^5 - 1) = (243 + 1)(243 - 1) = 244 \times 242$$

$$(b) = \text{حاصل ضرب} : (a_1 \times a_{10})^{\frac{n}{2}} = (1 \times 242)^5 = (242)^5$$

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(7)

$$a, aq, aq^2, \dots$$

$$aq^2 - aq = aq(1 - q)$$

$$a - aq = a(1 - q)$$

$$\frac{aq(1-q)}{a(1-q)} = q$$

$\Rightarrow$ نسبت آن با قدر نسبت دنبال
اولیه برابر

$$ب) S_n = a \left(\frac{q^n - 1}{q - 1} \right)$$

$$S_n = a, aq, aq^2, \dots, aq^{n-1}$$

⑥ - ثابت شد

طرف اول ضرب در q

$$q S_n = aq + aq^2 + \dots + aq^n$$

$$S_n - q S_n = (a + aq + \dots + aq^{n-1}) - (aq + aq^2 + \dots + aq^n)$$

$$\hookrightarrow S_n (1 - q) = a - aq^n \rightarrow S_n = \frac{a(1 - q^n)}{1 - q}$$

الف) $S_n = \frac{n}{2} (2a + (n-1)d)$

$$S_n = a + a + d + a + 2d + \dots + a_n$$

$$a_n = a_1 + (n-1)d$$

$$\begin{cases} S_n = a_1 + a_2 + a_3 + \dots + a_{n-1} + a_n \\ S_n = a_n + a_{n-1} + \dots + a_2 + a_1 \end{cases}$$

$$2S_n = (a_1 + a_n) + (a_2 + a_{n-1}) + \dots + (a_n + a_1)$$

طرفین را جمع
کنیم

$$\hookrightarrow 2S_n = n(a_1 + a_n)$$

طرفین را در 2 ضرب می‌کنیم

$$S_n = \frac{(a_1 + a_n)n}{2}$$