۱) الف)

$a_1 \times a q^9 = \frac{1}{2} \times 2^9 = 2^8 = 256$

ب)

$\frac{a_{17}}{a_{13}} = \frac{a_1 q^{16}}{a_1 q^{12}} = q^4 = 2^4 = 16$

ج)

$a_4 = a_1 q^3 = \frac{1}{2} \times 2^3 = 4 \qquad \frac{256}{4} = 64$

د)

$a_n = 128 = a_1 \times q^{n-1}, \quad a_1 = \frac{1}{2} \qquad 256 = q^{n-1} \qquad n-1 = 8 \qquad n = 9$

۲)

$a_5 = a_1 q^4 = 12$

$a_8 = a_1 q^7 = 96 \qquad \frac{a_1 q^7}{a_1 q^4} = \frac{96}{12} \qquad q^3 = 8 \quad q = 2 \qquad a_{10} = a_5 q^5 = 12 \times 32 = 384$

۳)

$a_1 \times a_2 \times a_3 \times a_4 \times a_5 = 243$

$\frac{x}{q^2} \times \frac{x}{q} \times x \times xq \times xq^2 = x^5 = 243 \qquad x = 3 = a_3 \qquad a_1 a_5 = a_3 a_3 = a_3^2 = 3 \times 3 = 9$

۴)

$4\sqrt{2} = 2^2 \times 2^{\frac{1}{2}} = 2^{2+\frac{1}{2}} = 2^{2.5} \qquad \frac{2^{2.5}}{2^a} = \frac{2^b}{2^{2.5}} \qquad 2^5 = 2^{a+b}$

$2^a, 2^{2.5}, 2^b \qquad a + b = 5 \qquad \frac{a+b}{2} = \frac{5}{2} = 2.5$

$b = 5 - a$

۵)

$a_3 = a_1 q^2 = x - 1 \qquad a_7 = a_1 q^6 = x \qquad a_{11} = a_1 q^{10} = x + 1$

$\frac{a_7}{a_3} = \frac{a_1 q^6}{a_1 q^2} = q^4 = \frac{x}{1-x} \qquad \frac{a_{11}}{a_7} = \frac{a_1 q^{10}}{a_1 q^6} = q^4 = \frac{x+1}{x}$

$\frac{x+1}{x} = \frac{x}{1-x} \qquad x^2 = (x+1)(1-x) \Rightarrow x^2 = 1 - x^2 \qquad 2x^2 = 1 \qquad x^2 = \frac{1}{2} \qquad x = \pm\sqrt{\frac{1}{2}}$

۶)

$a + aq^3 = 28 \qquad a(1+q^3) = 28 \qquad \frac{aq(1+q)}{a(1+q^3)} = \frac{12}{28} = \frac{3}{7} \qquad 7 + 7q = 3 + 3q^3$

$aq + aq^2 = 12 \qquad aq(1+q) = 12 \qquad 3q^2 - 10q + 3 = 0$

$q = \frac{10 \pm \sqrt{100 - 36}}{6} = \frac{10 \pm 8}{6} \qquad q = 3 \ \text{یا}\ q = \frac{1}{3}$

$q = 3 \longrightarrow a = \frac{12}{3(1+3)} = \frac{12}{12} = 1$ $\qquad$ بزرگترین $= 27$

$q = \frac{1}{3} \longrightarrow a = \frac{12}{\frac{1}{3}(1+\frac{1}{3})} = \frac{12}{\frac{4}{9}} = 27$

(۷

$a + aq + aq^2 = 14$

$a(1+q+q^2) = 14 \xrightarrow{a=\frac{3}{q}} \frac{3}{q}(1+q+q^2) = 14$

$3(1+q+q^2) = 14q$

$3 + 3q + 3q^2 = 14q$

$3q^2 - 11q + 3 = 0$

$q = \frac{11 \pm 5}{6}$

$q = 3 \;\text{یا}\; q = \frac{1}{3}$

$a \times aq \times aq^2 = a^3 q^3 = 27$

$aq = 3 \qquad a = \frac{3}{q}$

$q = 3 \rightarrow 1, 3, 9 \rightarrow a = 1$

$q = \frac{1}{3} \rightarrow 9, 3, 1 \rightarrow a = 9$

(۸

$S_{10} = a_1\left(\frac{q^{10}-1}{q-1}\right)$

$a_1 = 2 \quad q = 3$

$S_{10} = 2 \times \left(\frac{3^{10}-1}{3-1}\right) = 2\left(\frac{59049-1}{2}\right) = 59048$

$a_1 \times a_2 \times a_3 \times \cdots \times a_{10}$

$a_1^{10} \times q^{(0+1+2+\cdots+9)} = 2^{10} \times 3^{45}$

(۹

$b_n = a_{n+1} - a_n$

$aq^n - aq^{n-1} = a_1 q^{n-1}(q-1)$

$q_{\text{اولیه}} = q_{\text{نامیره}} \rightarrow$ هندسی است

$\frac{b_{n+1}}{b_n} = \frac{a_1 q^n (q-1)}{a_1 q^{n-1}(q-1)} = \frac{q^n}{q^{n-1}} = q^{n-n+1} = q^1 = q$ ← قدرنسبت $= q$

(۱۰ الف)

$a_1 = a \qquad a_2 = a + d \qquad a_3 = a + 2d \quad \cdots \quad a_n = a + (n-1)d$

$S_n = a + (a+d) + (a+2d) + \cdots + a + (n-1)d$

$S_n = (a + (n-1)d) + (a + (n-2)d) + \cdots + a$

$2S_n = n(2a + (n-1)d) \qquad S_n = \frac{n}{2}(2a + (n-1)d)$

(ب

$a_1 = a \qquad a_2 = aq \qquad a_3 = aq^2 \qquad a_n = aq^{n-1}$

$S_n = a + aq + aq^2 + \cdots + aq^{n-1}$

$qS_n = aq + aq^2 + \cdots + aq^n$

$(q-1)S_n = aq^n - a = a(q^n - 1)$

$S_n = a\left(\frac{q^n - 1}{q-1}\right)$