پرتو پرورش — کلاس : دهم ۹، دهم ۱۰ : ۱۸ و ۱۶:۲۰

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الف) $a=\frac{1}{2}$ $\quad q=2$ $\qquad a_{10}=aq^{9}=\frac{1}{2}\times 2^{9}=2^{8}=256$

ب) $\frac{a_{17}}{a_{13}}=q^{4}=2^{4}$

ج) $a_{4}\times a_{10}=a_{7}^{2}\rightarrow$ واسطه $: a_{7}=\frac{1}{2}\times 2^{6}=2^{5}$

د) $128=\frac{1}{2}\times 2^{n-1}$ $\qquad 2^{n-1}=256=2^{8}$ $\qquad n=7$

$\frac{a_{8}}{a_{5}}=q^{3}\longrightarrow \frac{96}{12}=8=2^{3}\Rightarrow q=2$

$a_{10}=a_{8}\times q^{2}=96\times 4=384$

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الف) $a_{1}\times a_{2}\times\dots\times a_{5}=P_{5}=a_{3}^{5}\Rightarrow a_{3}=\sqrt[5]{P_{5}}=\sqrt[5]{243}$

ب) $a_{1}\times a_{5}=a_{3}^{2}=\sqrt[5]{243}^{2}=$

$2^{a}, \frac{4\sqrt{2}}{32}, 2^{b}\longrightarrow 2^{a}, \sqrt[5]{2}, 2^{b}$ $\qquad 2^{a}\times 2^{b}=2^{5}$

$a+b=5$ واسطه حسابی $=\frac{a+b}{2}=\frac{5}{2}$

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$a_{3}=1-x$ $\qquad a_{7}=x$ $\qquad a_{11}=x+1$

$a_{3}\times a_{11}=a_{7}\times a_{7}\Rightarrow (1-x)(1+x)=x^{2}$ $\qquad 1^{2}-x^{2}=x^{2}$

$2x^{2}=1$ $\qquad x=\pm\sqrt{\frac{1}{2}}$

$$\left.\begin{array}{l} a_1 + a_4 = 28 \\ a_2 + a_3 = 12 \end{array}\right\} \Rightarrow S_4 = 40 = a_1\left(\frac{q^4-1}{q-1}\right)$$ ۶

$$a_1 q + a_1 q^2 = 12 \longrightarrow a = \frac{12}{q(1+q)}$$

$$\Rightarrow 40 = \frac{12}{q(q+1)} \times \frac{(q^2+1)\cancel{(q-1)}\cancel{(q+1)}}{(q-1)}$$

$$40q = 12q^2 + 12 \longrightarrow 3q^2 + 3 - 10q = 0$$

$$\begin{cases} q = 3 \longrightarrow a = 1 & 1, 3, 9, \underline{\underline{27}} \\ q = \frac{1}{3} \longrightarrow a = 27 & \underline{\underline{27}}, 9, 3, 1 \end{cases} \longrightarrow \boxed{27}$$

$$a_1 + a_2 + a_3 = 13 \qquad \frac{x}{q} + x + qx$$

$$a_1 \cdot a_2 \cdot a_3 = 27$$

$$\frac{x}{\cancel{q}} \cdot x \cdot \cancel{q} x = x^3 \to x = 3, \qquad \frac{3}{q} + 3q = 10$$

$$3 + 3q^2 - 10q = 0$$

$$\begin{cases} q = 3 & \dots \quad 1, 3, 9 \\ q = \frac{1}{3} & 9, 3, 1 \end{cases}$$

الف) $q = 3 \quad a = 2$

$$S_{10} = 2\left(\frac{3^{10}-1}{3-1}\right) = 3^{10} - 1$$

ب) $a_{10} = 2 \times 3^9$

$$P_n = \sqrt{(a_1 \cdot a_n)^n} \qquad P_{10} = \sqrt{(4 \times 3^9)^{10}} = (4 \times 3^9)^5$$

$$a_2 - a_1 = aq - a = a(q-1)$$

$\times q$

$$a_3 - a_2 = aq^2 - aq = aq(q-1)$$

$\times q$

$$a_4 - a_3 = aq^3 - aq^2 = aq^2(q-1)$$

$$a_n - a_{n-1} = aq^{n-1} - aq^{n-2} = aq^{n-2}(q-1)$$

الف)

$$a_1 + a_2 + \dots + a_n = S_n$$

$$a_1 + a_1 + d + a_1 + 2d + \dots + a_1 + (n-1)d =$$

$$na_1 + \underbrace{d + 2d + 3d + \dots + (n-1)d}_{d(1+2+\dots+n-1) \to \frac{(n-1)n}{2}} =$$

$$na_1 + \frac{(n-1)n}{2} d = \frac{n}{2}\left[2a + (n-1)d\right] = S_n$$

حکم ✓

ب) ۱۰

$$S_n = a_1 + a_2 + \dots + a_n \Rightarrow$$

$$\left[S_n = a_1 + aq + aq^2 + \dots + aq^{n-1}\right] \times q$$

$$S_n \times q = a_1 q + aq^2 + aq^3 + \dots + aq^n$$

$$\underbrace{a_2 \quad a_3 + \dots + a_n}_{S_n - a} + aq^n$$

$$S_n q = S_n - a_1 + aq^n \longrightarrow S_n(q-1) = a_1(q^n - 1) \to$$

حکم ✓