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الف) $a_n = \frac{1}{2} \times 2^{n-1} \rightarrow a_{10} = \frac{1}{2} \times 2^9 = 2^8$

ب) $\frac{a_{17}}{a_{13}} = \frac{\frac{1}{2} \times 2^{16}}{\frac{1}{2} \times 2^{12}} = 2^4 = 16$ ✓

ج) $\text{وسط} = \sqrt{a_4 \times a_{10}} = \sqrt{2^2 \times 2^8} = 2^5$

د) $\frac{1}{2} \times 2^{n-1} = 128 \Rightarrow 2^{n-2} = 2^7 \Rightarrow n-2 = 7 \Rightarrow n = 9$

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$\left.\begin{matrix} a_8 = 96 \\ a_5 = 12 \end{matrix}\right\} \rightarrow \frac{a_8}{a_5} = \frac{a q^7}{a q^4} = \frac{96}{12} \Rightarrow q^3 = 8 \Rightarrow q = 2$

$a q^4 = 12 \Rightarrow a = \frac{3}{4} \rightarrow a_{10} = a q^9 = \frac{3}{4} \times 2^9 = 3 \times 2^7$ ✓

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الف) $a_1 \times a_2 \times a_3 \times a_4 \times a_5 = 243 \Rightarrow \frac{a_3}{q^2} \times \frac{a_3}{q} \times a_3 \times a_3 q \times a_3 q^2 = 243 \Rightarrow a_3^5 = 243 \Rightarrow a_3 = 3$ ✓

ب) $a_1 \times a_5 = a_2 \times a_4 = 3 \times 3 = 9$

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$a_1 \times a_4 = a_2 \times a_3 \Rightarrow 2^a \times 2^b = 4\sqrt{2} \times 4\sqrt{2} \Rightarrow 2^{a+b} = 2^3 \Rightarrow a+b = 3$

$\text{وسط} = \frac{a+b}{2} = \frac{3}{2} = 1/5$ ✓

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$a_3 \times a_{11} = a_7 \times a_7 \Rightarrow (x+1)(1-x) = x^2 \Rightarrow 1 - x^2 = x^2 \Rightarrow 1 = 2x^2 \Rightarrow x^2 = \frac{1}{2} \Rightarrow x = \pm\sqrt{\frac{1}{2}}$ ✓ ۱۱۷۵

فقط $\sqrt{\frac{1}{2}}$ درست

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$a_1 + a_4 = 28 \Rightarrow a(1+r^3) = 28$

$a_2 + a_3 = 12 \rightarrow a r (1+r) = 12$

$\frac{a(1+r^3)}{ar(1+r)} = \frac{(1+r)(1-r+r^2)}{r(1+r)} = \frac{7}{3} \Rightarrow 3 - 3r + 3r^2 = 7r$

$\Rightarrow 3r^2 - 10r + 3 = 0$

$\Rightarrow r^2 - 10r + 9 = 0$

$\Rightarrow (r-1)(r-9) = 0$

$\Rightarrow r = \frac{1}{3}, \quad r = 3$

$r = 3 \rightarrow a(1+27) = 28 \Rightarrow a = 1$ ✓

$r = \frac{1}{3} \quad a\left(1 + \frac{1}{27}\right) = 12 \Rightarrow a = \frac{81}{7}$ ✗

$a_1 + a_2 = \frac{81}{7} + \frac{81}{7} \times \frac{1}{27} = \frac{84}{7} \neq 12$

$a_1 = 1, \quad a_2 = 3, \quad a_3 = 9, \quad a_4 = 27$

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$a_1 \times a_2 \times a_3 = 27 \Rightarrow \frac{a_2}{q} \times a_2 \times a_2 q = 27 \Rightarrow a_2 = 3$

$a_1 + a_2 + a_3 = 13 \rightarrow \frac{3}{q} + 3 + 3q = 13 \Rightarrow \frac{3}{q} + 3q = 10 \Rightarrow \frac{3 + 3q^2}{q} = 10 \Rightarrow 3 + 3q^2 = 10q$

$\Rightarrow 3q^2 - 10q + 3 = 0 \Rightarrow q^2 - 10q + 9 = 0 \Rightarrow (q-9)(q-1) = 0 \Rightarrow q = 9, \quad q = \frac{1}{3}$

$a_n = 1, 3, 9 \quad - \quad a_n = 9, 3, 1$

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الف) $S_{10} = a\left(\frac{q^{10}-1}{q-1}\right) = 2\left(\frac{3^{10}-1}{2}\right) = 3^{10} - 1$

ب) $P_{10} = \sqrt{(a_1 a_{10})^{10}} = (a_1 a_{10})^5 = (2 \times 2 \times 3^9)^5 = (2^5 \times 3^{45})$

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$a, aq, aq^2, \ldots aq^n \rightarrow a - \frac{a}{q}, \quad aq - a, \quad aq^2 - aq, \ldots, aq^n - aq^{n-1}$

$\frac{a}{q}(q-1), \quad a(q-1), \quad aq(q-1), \ldots, aq^{n-1}(q-1)$

$\times q \qquad \times q \qquad \times q \qquad \times q$

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سوال ۱۵

الف) $a + a+d + a+2d + \dots + a+(n-1)d = na + \frac{n(n-1)}{2} \times d = na + \frac{n}{2} \times (n-1)d$

$= \frac{n}{2}(2a+(n-1)d)$

ب) $\left.\begin{array}{l} S_n = a + ar + ar^2 + \dots + ar^{n-1} \\ rS_n = ar + ar^2 + ar^3 + \dots + ar^n \end{array}\right\} \rightarrow S_n - rS_n = a - ar^n \rightarrow S_n(1-r) = a(1-r^n)$

$S_n = a\left(\frac{1-r^n}{1-r}\right)$

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