به نام خدا

سارینا زارع — دهم جمعه A — تکلیف شماره ۱۲ — ۰۴/۷/۲۸

①

الف) $t_n = 4n + 1$

ب) $t_{10} = 10 \times 4 + 1 = 41$

②

$t_n = 6, 10, 14, 18, \dots \rightarrow t_n = 4n + 2 \rightarrow t_n = t_1 + (n-1)d \Rightarrow t_n = 6 + (n-1)4$

الف) $S_{10} = \frac{10}{2}\left(2 \times 6 + (10-1) \times 4\right) = 48 \times 5 = 240$

ب) $4 \times 8 = 32 \longrightarrow t_{30} + t_{31} + t_{32} = 2t_{31} + t_{31} = 3t_{31}$

$t_{31} = 6 + 30 \times 4 = 126$

③

$t_n = 1 + \sqrt{3},\ 2,\ 3 - \sqrt{3}$

$t_{25} - t_{23} = (t_1 + (25-1)d) - (t_1 + (23-1)d) \rightarrow t_1 + 24d - t_1 - 22d = 24d - 22d = \boxed{2d}$ ★

$t_3 - t_1 \Rightarrow (3 - \sqrt{3}) - (1 + \sqrt{3}) = \boxed{2 - 2\sqrt{3}}$

↳ $(t_1 + 2d) - (t_1) = \boxed{2d}$ ★

④

$a_n = 5^{2x},\ 3 \times 25^{x},\ 5^{y} \rightarrow (3 \times 5^{2x}) - 5^{2x} = 5^{y} - (3 \times 5^{2x})$

$(5^2)^x \rightarrow 5^{2x}$

↳ $(3-1)5^{2x} = 5^{y} - 3 \times 5^{2x} \rightarrow 5^{y} = 5^{2x}(2+3) = 5^{2x} \times 5 = \boxed{5^{2x+1}}$

$5^{y} = 5^{2x+1} \rightarrow y = 2x + 1$

$b_n = x,\ 2,\ y$

$xy = ? \rightarrow 3 \times 1 = \boxed{3}$

↳ $b_2 - b_1 = b_3 - b_2 \Rightarrow 2 - x = y - 2 \xrightarrow{y = 2x+1} 2 - x = (2x+1) - 2 \rightarrow 2 - x = 2x - 1$

↳ $3x = 3 \rightarrow \boxed{x = 1}$

$y = 2x + 1 \xrightarrow{x=1} 2(1) + 1 = \boxed{3 = y}$

⑤

$a_n = 2x - 4,\ 2x - 1,\ 6x$ $\qquad$ $a_4 = ? \rightarrow a_4 = -3 + (4-1) \times 3 = \boxed{6}$

↳ $6x - (2x - 1) = (2x - 1) - (2x - 4) \rightarrow 6x - 2x + 1 = 2x - 1 - 2x + 4 \rightarrow 4x = 2 \rightarrow x = \frac{1}{2}$

↳ $a_n = -3, 0, 3, \dots \rightarrow 3 - 0 = 3 = d$

⑥

$a_n = 3, 5, 7, \dots$ $\qquad$ $b_n = 2, 5, 8, \dots$ $\qquad$ $a_{20} = 20 \times 2 + 1 = 41$

↳ $2n + 1 = a_n \rightarrow d = 2$ $\qquad$ ↳ $b_n = 3n - 1 \rightarrow d = 3$ $\qquad$ $b_{20} = 3 \times 20 - 1 = 59$

$3 \times 2 = 6$ ← جملات مشترک ۶تا۶تا تکرار می‌شوند

اولین جمله مشترک = ۵ ← جملات مشترک = $5, 11, 17, 23, 29, 35, 41, \dots$

تعداد جملات مشترک $\Rightarrow \frac{41 - 5}{6} + 1 = \boxed{7}$

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$a_1+a_2+a_3=21 \rightarrow 3a_2=21 \rightarrow a_2=7$

$a_1+a_3+a_5=15 \rightarrow 3a_3=15 \rightarrow a_3=5$

$a_3-a_2+a_1 \Rightarrow 5-7+(-21)=\frac{7}{-21}=-\frac{1}{3}$

$d=3-7=28$

$a_1=7-28=-21$

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$a_1+a_2+a_3=15 \rightarrow 3a_2=15 \rightarrow a_2=5 \Longrightarrow a_1=5-d$

$a_4+a_5=25 \rightarrow (a_1+3d)+(a_1+4d)=2a_1+7d=25$

$2(5-d)+7d=25 \rightarrow 5d=15 \rightarrow d=\boxed{3}$, $a=5-3=\boxed{2}$

$a_n=2+(n-1)3 \rightarrow a_{10}=2+(10-1)3 \Rightarrow 27+2=29$

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$S_9=9S_3$ $\qquad \frac{a_{20}}{a_7}=?$

$S_9=\frac{9}{2}(2a+8d)$ ✱

$S_3=\frac{3}{2}(2a+2d)$ ✱

$\frac{\frac{9}{2}(2a+8d)}{9(a+4d)}=9\times\frac{\frac{3}{2}(2a+2d)}{9\times 3(a+d)} \rightarrow a+4d=3d+3d$

$d=2a$

$a_{20}=a+19d \Rightarrow a+19(2a)=39a$

$a_7=a+9d \Rightarrow a+9(2a)=19a$

$\frac{39a}{19a}=\boxed{3}$

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$a_1=11$, $a_7=35 \rightarrow d=\frac{35-11}{7-1}=\frac{24}{6}=\boxed{4}$

$a_4=11+3d \Rightarrow 11+12=23$

$d'=\frac{13-31}{k+1}=\frac{-28}{k+1}$

$a'_4=31+3d' \Rightarrow 31+3\left(\frac{-28}{k+1}\right) \Rightarrow 31-\frac{78}{k+1}=23 \rightarrow \frac{78}{k+1}=18 \Longrightarrow 18k+18=78$

$k=4$

۴ واسطه