نام: سنا فاضلی   دهم جعبه ها

تکلیف: ۱۲

۱- الف) $t_n = 4n+1$ (۴ از قدر نسبت به دست آمده)

ب) $t_{10} = 4\times10+1 = 41$

۲- الف) $S_n = \frac{n}{2}(a_1+a_n) \to \cancel{\frac{10}{2}}^{5}(\overset{48}{\cancel{6+42}}) = 240$

$t_n = 4n+2 \to t_{10} = 40+2 = 42$

ب) $4\times8 = 32 \to$ جملات : $29, 30, 31, 32$

$S_{28} = (6+114)\times14 = 1680$ $S_{32} = (6+130)\times16 = 2176$

$t_{28} = 4\times28+2 = 114$ $t_{32} = 4\times32+2 = 130$

$2176 - 1680 = 496$

۳- $t_n = (1-\sqrt{3})n + 2\sqrt{3}$

$d = 2-(1+\sqrt{3}) = 1-\sqrt{3}$

$t_{25} = 25(1-\sqrt{3}) + 2\sqrt{3} = 25 - 25\sqrt{3} + 2\sqrt{3} = 25 - 23\sqrt{3}$

$t_{23} = 23(1-\sqrt{3}) + 2\sqrt{3} = 23 - 23\sqrt{3} + 2\sqrt{3} = 23 - 21\sqrt{3}$

$t_{25} - t_{23} = 25 - 23\sqrt{3} - 23 + 21\sqrt{3} = 2 - 2\sqrt{3}$

(می توانستیم از همان ابتدا بگوئیم جمله ی ۲۵ ام با ۲۳ ام ۲d فاصله دارد پس $2\times(1-\sqrt{3})$ پاسخ است)

۴- $*\,5^{2x} = 25^x \to \frac{5^y + 2\,25^x}{2} = 3\times25^x \to 5^y + 25^x = 6\times25^x \to 6\times25^x - 25^x = 5^y \to 25^x(\cancel{6-1})_{5} = 5^y$

$\to 5^{2x}\times5 = 5^y \to 2x+1 = y \xrightarrow{\text{حدس}} x=1 \quad y=3 \to xy = 3\times1 = \boxed{3}$

$5^2,\ 5^2\times3,\ 5^3 \to 25, 75, 125$

از دنباله ۲ ← $\begin{cases} x+y=4 \\ 2x+1=y \end{cases} \to y=3,\ x=1$

۵- $\frac{6x+2x-4}{2} = 2x-1 \to 8x-4 = 4x-2 \to 4x = 2 \to x = \frac{1}{2}$

$t_n = -3, 0, 3 \to t_n = 3n-6 \to t_4 = 3\times4-6 = 6$

۶- $b_n = \underset{+3}{2, 5, 8}$ ، $3n-1$ $a_n = \underset{+2}{3, 5, 7}$ ، $2n+3$ دنباله مشترک $t_n = 6n-1$

ک.م.م ها $= 6$

$\to 6n-1 \leq 43$

$6n \leq 44$

$n \leq \frac{44}{6}$ $n \leq 7.3 \to$ ۷ جمله مشترک

$a_{20} = 2\times20+3 = 43$

$b_{20} = 3\times20-1 = 59$ $\Rightarrow 43 < 59$

۷- $a_1 + \underbrace{a_1+d}_{a_2} + \underbrace{a_1+2d}_{a_3} = 21 \to 3a_1+3d = 21 \to \overset{-21}{\cancel{a_1}} + \overset{28}{\cancel{d}} = 7$

$a_1 + \underbrace{a_1+2d}_{a_3} + \underbrace{a_1+4d}_{a_5} = 105 \to 3a_1+6d = 105 \Rightarrow \begin{cases} 3a_1+3d = 21 \\ 3a_1+6d = 105 \end{cases} \Rightarrow 3d = 84 \to d = 28$

$a_3 - a_2 + a_1 = a_1+2d - a_1 - d + a_1 = a_1+d \Rightarrow \frac{a_3-a_2+a_1}{a_1} = \frac{a_1+d}{a_1} = \frac{7}{-21} = -\frac{1}{3}$

۸-

$a-2d + a-d + a + a+d + a+2d = 40 \rightarrow 5a = 40 \rightarrow a = 8$

$3a - 3d = 15 \rightarrow 24 - 9 = 15 \rightarrow d = \boxed{3}$

$2, 5, 8, 11, \dots \rightarrow t_n = 3n - 1 \rightarrow t_{10} = 30 - 1 = \boxed{29}$

۹-

$\frac{9}{2}\left(2a_1 + (n-1)d\right) = \frac{3\times 9}{2}\left(2a_1 + (n-1)d\right)$

$\frac{9}{2}\left(2a_1 + 8d\right) = \frac{27}{2}\left(2a_1 + 2d\right) \rightarrow 9\left(2a_1 + 8d\right) = 27\left(2a_1 + 2d\right) \rightarrow 18a + 72d = 54a_1 + 54d$

$\rightarrow 36a_1 = 18d \rightarrow 2a_1 = d \rightarrow \frac{a_{20}}{a_7} = \frac{a_1 + 19d}{a_1 + 6d} = \frac{a_1 + 38a_1}{a_1 + 12a_1} = \frac{39a_1}{13a_1} = \boxed{3}$

۱۰-

$7 - 1 = 6 \rightarrow 6d = 35 - 11 = 24 \rightarrow d = \boxed{4}$

$\rightarrow 11, 15, 19, \underline{23}$

$\rightarrow 38, \dots, 13 \rightarrow \frac{13-38}{n+1} = -5 \rightarrow n+1 = 5 \rightarrow n = \boxed{4} \rightarrow$ ۴ جمله درج باید بشود

$23 - 38 = -15 \rightarrow 3d = -15 \rightarrow d = -5$