۱) الف)

$5 \to 9, 13, 17, \dots$ (+۴ +۴ +۴) $d=4$ $a_1=5$ $a_n = 4n+1$

ب)

$a_{10} = 4\times10+1 = 41$

۲) الف)

$9, 10, 14, 18, \dots$ (+۴ +۴ +۴) $d=4$ $a_1=9$ $t_n = 4n+2$ $a_{10} = 4\times10+2 = 42$

$S_{10} = 5(9+42) = 5\times48 = 240$

ب)

$4\times8 = 32 \xrightarrow{\text{آمین}} a_{32}$ $\quad t_{19}+t_{30}+t_{21}+t_{32} = 2t_{30}+t_{22}$

$2(4\times30+2) + (4\times22+2) = 2\times122 + 120 = 364$

۳) $1+\sqrt{2}, 2, 3-\sqrt{2}, \dots$ (+$1-\sqrt{2}$, +$1-\sqrt{2}$)

$d = 1-\sqrt{2}$ $\quad a_1 = 1+\sqrt{2}$ $\quad t_n = (1-\sqrt{2})n + 2\sqrt{2}$

$t_{22} = (1-\sqrt{2})22 + 2\sqrt{2}$
$= 22 - 22\sqrt{2} + 2\sqrt{2}$
$= 22 - 21\sqrt{2}$

$t_5 = 25(1-\sqrt{2}) + 2\sqrt{2}$
$= 25 - 25\sqrt{2} + 2\sqrt{2}$
$= 25 - 23\sqrt{2}$

$t_{25} - t_{22} = 25 - 22\sqrt{2} - 22 + 21\sqrt{2}$
$= 2 - 2\sqrt{2}$

۴) $a_n = 5^{2x}, 3\times25^x, 5^y$

$3\times5^{2x} - 5^{2x} = 5^y - 3\times5^{2x}$

$5^y - 3\times5^{2x} = 2\times5^{2x}$

$5^y = 5\times5^{2x} = 5^{2x+1}$

$b_n = x, 2, y$

$2-x = y-2$

$y = 4-x$ $\quad 4-x = 2x+1$ $\quad xy = 1\times3 = 3$

$3 = 3x$

$x = 1$

۵) $2x-4, 2x-1, 4x, \dots$ $\quad 4\left(\frac{1}{2}\right)-4, 2\left(\frac{1}{2}\right)-1, 4\left(\frac{1}{2}\right), \dots$

$-3, 0, 2$ $\quad d=3$ $\quad a_1=-3$ $\quad a_n = 3n-6$

$a_6 = 3\times6-6 = 18-6 = 12$

$4x-2-2x+4 = 4x-2x+1$

$3 = 4x+1$ $\quad 2x=2$ $\quad x=\frac{1}{2}$

۶) $3, 5, 7, \dots$ $\quad d=2$ $\quad a_1=3$ $\quad a_n = 2n+1$

$2, 5, 8, \dots$ $\quad d=3$ $\quad b_1=2$ $\quad b_n = 3m-1$

$2n+1 = 3m-1$

$2n = 3m-2$

$n = \frac{3m-2}{2} \le 20$

$m \le 14$

$m = 2, 6, 4, 8, 10, 12, 14$

$m=2 \to 3\times2-1 = 5$
$m=6 \to 3\times6-1 = 17$
$m=4 \to 3\times4-1 = 11$
$m=8 \to 3\times8-1 = 23$
$m=10 \to 3\times10-1 = 29$
$m=12 \to 3\times12-1 = 35$
$m=14 \to 3\times14-1 = 41$

۷) $a_1+a_2+a_3 = 21$ $\quad a_1+a_1+d+a_1+2d = 3a_1+3d = 21$ $\quad a_1+d=7$

$a_1+a_4+a_5 = 105$ $\quad a_1+a_1+3d+a_1+4d = 3a_1+9d = 105$ $\quad a_1+3d=35$

$d=28$ $\quad a_1=-21$

$a_3 - a_2 + a_1 = a_1+2d-a_1-d+a_1 = a_1+d = 28-21 = 7$

$\frac{a_3-a_2+a_1}{a_1} = \frac{7}{-21} = -\frac{1}{3}$

$a_1+a_2+a_3 = a_1+a_1+d+a_1+2d = 3a_1+3d = 15 \quad a_1+d=5$ (۸

$a_4+a_5 = a_1+3d+a_1+4d = 2a_1+7d = 25$

$\begin{cases} a_1+d=5 \\ 2a_1+7d=25 \end{cases} \times 2 \quad \begin{cases} 2a_1+2d=10 \\ 2a_1+7d=25 \end{cases}$

$5d=15 \quad d=3$

$a_1=2$

$2, 5, 8, \dots \qquad a_n = 3n-1$

$a_{10} = 3\times 10 - 1 = 29$

$S_9 = \frac{9}{2}(2a+8d)$ (۹

$\frac{9}{2}(2a+8d) = \frac{27}{2}(2a+2d)$

$9S_3 = 9\times\frac{3}{2}(2a+2d)$

$9a+36d = 27a+27d$

$9d = 18a \quad d=2a$

$a_{20} = a_1+19d = a+19(2a) = a+38a = 39a$

$\frac{a_{20}}{a_7} = \frac{39d}{13d} = 3$

$a_7 = a+9d = a+18a = 13a$

$a_1 = 11 \qquad a_7 = a_1+9d = 35$ (۱۰

$11+9d = 35 \quad 9d = 24 \quad d = 4 \qquad a_n = 4n+7$

$a_4 = 4\times 4+7 = 23$

$a_1 = 38 \qquad a_n = 13$

$d = \frac{13-38}{n-1} = \frac{-25}{n-1}$

$a_4 = 38+3d = 38-\frac{75}{n-1}$

$38-\frac{75}{n-1} = 23$

$\frac{75}{n-1} = 15$

$n-1 = \frac{75}{15} = 5 \qquad n=6$

$n-2 = 6-2 = 4$

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جملات وسط