

الف) $y = \sqrt{\frac{x^2-4x}{x^2-x}} = y \Rightarrow \frac{x^2-4x}{x^2-x} \geq 0, x^2-x \neq 0 \Rightarrow \textcircled{1}: x^2-4x=0 \rightarrow x(x-4)=0 \rightarrow \begin{matrix} x=0 \\ x=4 \end{matrix}$
 $\textcircled{2}: x^2-x=0 \rightarrow x=0, x=1$
 $\textcircled{3}: x^2-4x \neq 0 \rightarrow x(x-4) \neq 0 \rightarrow x \neq 0, x \neq 4$
 $\textcircled{4}: x^2-x \neq 0 \rightarrow x(x-1) \neq 0 \rightarrow x \neq 0, x \neq 1$
 $\textcircled{5}: \frac{x^2-4x}{x^2-x} \geq 0 \Rightarrow \frac{-1 \quad 0 \quad 4}{-+ \quad + \quad - \quad -+} \Rightarrow [4, +\infty) \cup (0, 1) = Df$
 ب) $y = \sqrt{\frac{x^2-x^3}{x^2-x}} = y \Rightarrow \frac{x^2-x^3}{x^2-x} \geq 0, x^2-x \neq 0 \Rightarrow \textcircled{1}: x^2-x^3=0 \rightarrow x^2(x-1)=0 \rightarrow \begin{matrix} x=0 \\ x=1 \end{matrix}$
 $\textcircled{2}: x^2-x=0 \rightarrow x=0, x=1$
 $\textcircled{3}: x^2-x^3 \neq 0 \rightarrow x^2(x-1) \neq 0 \rightarrow x \neq 0, x \neq 1$
 $\textcircled{4}: x^2-x \neq 0 \rightarrow x(x-1) \neq 0 \rightarrow x \neq 0, x \neq 1$
 $\textcircled{5}: \frac{x^2-x^3}{x^2-x} \geq 0 \Rightarrow \frac{-1 \quad 0 \quad 1}{-+ \quad + \quad - \quad -+} \Rightarrow (1, +\infty) \cup (0, 1) \cup (-1, 0) \cup (-1, -\infty) = Df$

الف) $y = \frac{x+1}{\sqrt{x+|x|}} \Rightarrow x+|x| \geq 0, \sqrt{x+|x|} \neq 0 \Rightarrow x+|x| > 0 \Rightarrow |x| > -x \Rightarrow \begin{matrix} \rightarrow 0, x \\ \rightarrow +\sqrt{x} \\ \rightarrow -x \end{matrix}$
 $\Rightarrow Df = \mathbb{R}^+$
 ب) $y = \frac{1}{\sqrt{x|x|}} \Rightarrow x|x| \geq 0, \sqrt{x|x|} \neq 0 \Rightarrow x|x| > 0 \Rightarrow \begin{matrix} 0, x \\ +\sqrt{x} \\ -x \end{matrix} \Rightarrow Df = \mathbb{R}^+$

الف) $y = \sqrt{2-[x]} \Rightarrow 2-[x] \geq 0 \Rightarrow [x] \leq 2 \Rightarrow x < 3 \Rightarrow Df = (-\infty, 3)$
 ب) $y = \frac{1}{\sqrt{2-[x]}} \Rightarrow 2-[x] \geq 0, \sqrt{2-[x]} \neq 0 \Rightarrow 2-[x] > 0 \Rightarrow [x] < 2$
 $\Rightarrow Df = (-\infty, 2)$

الف) $y = \sqrt{[x-\frac{1}{p}] + [x+\frac{p}{p}]} \Rightarrow [x-\frac{1}{p}] + [x+\frac{p}{p}] \geq 0 \Rightarrow [x+\frac{p}{p}] - 1 + [x+\frac{p}{p}] \geq 0$
 $\Rightarrow 2[x+\frac{p}{p}] \geq 1 \Rightarrow [x+\frac{p}{p}] \geq \frac{1}{2} \Rightarrow x+\frac{p}{p} \geq \frac{1}{2} \Rightarrow x \geq \frac{1}{p} \Rightarrow Df = [\frac{1}{p}, +\infty)$
 ب) $y = \sqrt{[x-\frac{1}{p}] + [-x+\frac{1}{p}]} \xrightarrow{\text{if}} \begin{cases} a \in \mathbb{Z} & [a] + [a] = 0 \checkmark \\ a \notin \mathbb{Z} & [a] + [-a] = -1 \times \end{cases} \Rightarrow x - \frac{1}{p} \in \mathbb{Z} \Rightarrow x = 2 + \frac{1}{p}$
 $\Rightarrow Df = \{x | x = k + \frac{1}{p}, k \in \mathbb{Z}\}$

الف) $y = \frac{r}{\sin x - 1} \Rightarrow \sin x - 1 \neq 0 \Rightarrow \sin x \neq 1 \Rightarrow x \neq 90^\circ \Rightarrow Df = \mathbb{R} - \{2k\pi + \frac{\pi}{2}\}$

ب) $\frac{r}{r \sin^2 x - 1} = y \Rightarrow r \sin^2 x - 1 \neq 0 \Rightarrow \sin^2 x \neq \frac{1}{r} \Rightarrow \sin x \neq \pm \frac{1}{\sqrt{r}}$
 $x \neq 45^\circ, 135^\circ, 225^\circ, 315^\circ \Rightarrow Df = \mathbb{R} - \{k\pi \pm \frac{\pi}{4}\}$

ا) $y = \sqrt{r \sin x - 1} \Rightarrow r \sin x - 1 \geq 0 \Rightarrow \sin x \geq \frac{1}{r} \Rightarrow Df = \left[rK\pi + \frac{\pi}{4}, rK\pi + \pi - \frac{\pi}{4} \right]$ ب) $y = \sqrt{\cos x - 1} \Rightarrow \cos x - 1 \geq 0 \Rightarrow \cos x \geq 1 \Rightarrow Df = \{rK\pi\}$	6
ا) $\cos y = x - r \Rightarrow -1 \leq x - r \leq 1 \Rightarrow r \leq x \leq r+1 \Rightarrow x \leq r+1, x \geq r$ $\Rightarrow -r-1 \leq x \leq -r \text{ , } x \leq -r \text{ , } x \geq r \Rightarrow Df = [-r-1, -r] \cup [r, r+1]$ ب) $y = \arcsin(x^r + rx + 1) \Rightarrow -1 \leq x^r + rx + 1 \leq 1 \Rightarrow \begin{cases} x^r + rx + 1 \leq 1 \\ x^r + rx + 1 \geq -1 \end{cases} \Rightarrow$ $\begin{cases} x^r + rx \leq 0 \Rightarrow x(x+r) \leq 0 \xrightarrow{x \geq 0} x \leq -r \xrightarrow{x \leq 0} x \geq -r \\ x^r + rx + 2 \geq 0 \Rightarrow (x+1)(x+r) \geq 0 \xrightarrow{x \geq -1} x \geq -r \xrightarrow{x \leq -1} x \leq -r \end{cases} \Rightarrow Df = [-r-1, -r] \cup [-1, 0] = Df$	7
ا) $y = \sqrt{r - \log_r(x-r)} \Rightarrow r - \log_r(x-r) \geq 0, x-r > 0 \Rightarrow \textcircled{1}: x > r$ $\textcircled{1}: \log_r(x-r) \leq r \rightarrow x-r \leq r^r \rightarrow x \leq r+1 \Rightarrow r < x \leq r+1 \Rightarrow Df = (r, r+1]$ ب) $y = \log(x^r - x) \Rightarrow x^r - x > 0 \rightarrow x(x^{r-1} - 1) > 0 \rightarrow \begin{matrix} x > 0 \\ x^{r-1} > 1 \end{matrix} \Rightarrow \begin{matrix} x > 0 \\ x > 1 \end{matrix} \Rightarrow Df = (1, +\infty) \cup (-1, 0)$	8
ا) $\frac{r}{x^n + 1} = y \Rightarrow x^n + 1 \neq 0 \Rightarrow x^n \neq -1 \Rightarrow Df = \mathbb{R}$ ب) $\frac{r}{x^n - 1} = y \Rightarrow x^n - 1 \neq 0 \Rightarrow x^n \neq 1 \Rightarrow x \neq 0 \Rightarrow Df = \mathbb{R} - \{0\}$ ج) $\frac{r}{x^n - r} = y \Rightarrow x^n - r \neq 0 \Rightarrow x^n \neq r \Rightarrow x \neq \log_r r \Rightarrow Df = \mathbb{R} - \{\log_r r\} \cup \mathbb{R} - \{\frac{1}{r}\}$ د) $\frac{r}{x^n - r} = y \Rightarrow x^n - r \neq 0 \Rightarrow x^n \neq r \Rightarrow x \neq \log_r r \Rightarrow Df = \mathbb{R} - \{\log_r r\}$	
ا) $y = (x+1)!$ $\Rightarrow x+1 \in \mathbb{N} \rightarrow x \in \mathbb{N} - 1 \Rightarrow x \in \frac{\mathbb{N}-1}{r} \Rightarrow$ $Df = \{x \mid x = \frac{K-1}{r}, K \in \mathbb{N}\}$ ب) $y = \left(\frac{rx-r}{rx-a} \right)!$ $\Rightarrow \frac{rx-r}{rx-a} \in \mathbb{N} \Rightarrow rx-r \in \mathbb{N} \cdot (rx-a) \Rightarrow$ $rx - rxw \in rx - aw \Rightarrow x(r-rw) \in rx - aw \Rightarrow x \in \frac{r-aw}{r-rw}$ $\Rightarrow Df = \{x \mid x = \frac{r-aw}{r-rw}, K \in \mathbb{N}\}$	10