

الف) $y = \sqrt{\frac{x^2-4x}{x^2-x}} \Rightarrow \frac{x^2-4x}{x^2-x} \geq 0, x^2-x \neq 0 \Rightarrow \textcircled{1}: x^2-4x=0 \Rightarrow x(x-4)=0 \Rightarrow \begin{matrix} x=0 \\ x=4 \end{matrix}$ $\textcircled{2}: x^2-x=0 \Rightarrow x=0, x=1$ $\textcircled{3}: x^2-4x \neq 0 \Rightarrow x(x-4) \neq 0 \Rightarrow x \neq 0, x \neq 4$ $\textcircled{4}: x^2-x \neq 0 \Rightarrow x(x-1) \neq 0 \Rightarrow x \neq 0, x \neq 1$ $\textcircled{5}: \frac{-1 \pm \sqrt{1+4}}{-2 \pm 1} = \frac{-1 \pm \sqrt{5}}{-2 \pm 1} \Rightarrow [4, +\infty) \cup (1, -1) = Df$ ب) $y = \sqrt{\frac{x^5-x^3}{x^2-x}} \Rightarrow \frac{x^5-x^3}{x^2-x} \geq 0, x^2-x \neq 0 \Rightarrow \textcircled{1}: x^5-x^3=0 \Rightarrow x^3(x^2-1)=0 \Rightarrow \begin{matrix} x=0 \\ x=\pm 1 \end{matrix}$ $\textcircled{2}: x^5-x^3 \neq 0 \Rightarrow x(x^2-1) \neq 0 \Rightarrow x \neq 0, x \neq \pm 1$ $\textcircled{3}: \frac{-1 \pm \sqrt{1+4}}{-2 \pm 1} = \frac{-1 \pm \sqrt{5}}{-2 \pm 1} \Rightarrow (1, +\infty) \cup (0, 1) \cup (-1, 0) \cup (-1, -\infty) = Df$	۱
الف) $y = \frac{x+1}{\sqrt{x+ x }} \Rightarrow x+ x \geq 0, \sqrt{x+ x } \neq 0 \Rightarrow x+ x > 0 \Rightarrow x > -x \Rightarrow \begin{matrix} 0 < x \\ +\sqrt{} \\ -x \end{matrix}$ $\Rightarrow Df = \mathbb{R}^+$ ب) $y = \frac{1}{\sqrt{x x }} \Rightarrow x x \geq 0, \sqrt{x x } \neq 0 \Rightarrow x x > 0 \Rightarrow \begin{matrix} 0 < x \\ +\sqrt{} \\ -x \end{matrix} \Rightarrow Df = \mathbb{R}^+$	۲
الف) $y = \sqrt{2-[x]} \Rightarrow 2-[x] \geq 0 \Rightarrow [x] \leq 2 \Rightarrow x < 3 \Rightarrow Df = (-\infty, 3)$ ب) $y = \frac{1}{\sqrt{2-[x]}} \Rightarrow 2-[x] \geq 0, \sqrt{2-[x]} \neq 0 \Rightarrow 2-[x] > 0 \Rightarrow [x] < 2$ $\Rightarrow Df = (-\infty, 2)$	۳
الف) $y = \sqrt{[x-\frac{1}{p}] + [x+\frac{p}{p}]} \Rightarrow [x-\frac{1}{p}] + [x+\frac{p}{p}] \geq 0 \Rightarrow [x+\frac{p}{p}] - 1 + [x+\frac{p}{p}] \geq 0$ $\Rightarrow 2[x+\frac{p}{p}] \geq 1 \Rightarrow [x+\frac{p}{p}] \geq \frac{1}{2} \Rightarrow x+\frac{p}{p} \geq \frac{1}{2} \Rightarrow x \geq \frac{1}{p} \Rightarrow Df = [\frac{1}{p}, +\infty)$ ب) $y = \sqrt{[x-\frac{1}{p}] + [-x+\frac{1}{p}]} \xrightarrow{\text{if}} \begin{cases} a \in \mathbb{Z} & [a] + [a] = 0 \checkmark \\ a \notin \mathbb{Z} & [a] + [-a] = -1 \times \end{cases} \Rightarrow x - \frac{1}{p} \in \mathbb{Z} \Rightarrow x = z + \frac{1}{p}$ $\Rightarrow Df = \{x x = k + \frac{1}{p}, k \in \mathbb{Z}\}$	۴
الف) $y = \frac{r}{\sin x - 1} \Rightarrow \sin x - 1 \neq 0 \Rightarrow \sin x \neq 1 \Rightarrow x \neq 90^\circ \Rightarrow Df = \mathbb{R} - \{2k\pi + \frac{\pi}{2}\}$ ب) $\frac{r}{r \sin^2 x - 1} = y \Rightarrow r \sin^2 x - 1 \neq 0 \Rightarrow \sin^2 x \neq \frac{1}{r} \Rightarrow \sin x \neq \pm \frac{1}{\sqrt{r}} \Rightarrow$ $x \neq 45^\circ, 135^\circ, 225^\circ, 315^\circ \Rightarrow Df = \mathbb{R} - \{k\pi \pm \frac{\pi}{4}\}$	۵

$$\text{أ) } y = \sqrt{r \sin x - 1} \Rightarrow r \sin x - 1 \geq 0 \Rightarrow \sin x \geq \frac{1}{r} \Rightarrow Df = \left[rK\pi + \frac{\pi}{4}, rK\pi + \pi - \frac{\pi}{4} \right]$$

$$\text{ب) } y = \sqrt{\cos x - 1} \Rightarrow \cos x - 1 \geq 0 \Rightarrow \cos x \geq 1 \Rightarrow Df = \{rK\pi\}$$

Ⓟ ✓

$$\text{أ) } \cos y = |x| - r \Rightarrow -1 \leq |x| - r \leq 1 \Rightarrow r \leq |x| \leq r+1 \Rightarrow |x| \leq r+1, |x| \geq r \\ \Rightarrow -r-1 \leq x \leq r+1, x \leq -r, x \geq r \Rightarrow Df = [-r-1, -r] \cup [r, r+1]$$

$$\text{ب) } y = \arcsin(x^r + rx + 1) \Rightarrow -1 \leq x^r + rx + 1 \leq 1 \Rightarrow \begin{cases} x^r + rx + 1 \leq 1 \\ x^r + rx + 1 \geq -1 \end{cases} \Rightarrow \\ \begin{cases} x^r + rx \leq 0 \Rightarrow x(x+r) \leq 0 \xrightarrow{x \geq 0} x \leq -r \xrightarrow{x \leq 0} x \geq -r \\ x^r + rx + 2 \geq 0 \Rightarrow (x+1)(x+r) \geq 0 \xrightarrow{x \geq -1} x \geq -r \xrightarrow{x \leq -1} x \leq -r \end{cases} \Rightarrow [-r, -r] \cup [-1, 0] = Df$$

$$\text{أ) } y = \sqrt{r - \log_r(x-r)} \Rightarrow r - \log_r(x-r) \geq 0, x-r > 0 \Rightarrow \text{Ⓟ: } x > r$$

$$\text{Ⓛ: } \log_r(x-r) \leq r \rightarrow x-r \leq r^r \rightarrow x \leq r+1 \Rightarrow r < x \leq r+1 \Rightarrow Df = (r, r+1]$$

$$\text{ب) } y = \log(x^r - x) \Rightarrow x^r - x > 0 \rightarrow x(x^{r-1} - 1) > 0 \rightarrow \begin{cases} x > 0 \\ x^{r-1} > 1 \end{cases} \Rightarrow \begin{cases} x > 0 \\ x \geq 1 \end{cases} \Rightarrow Df = (1, +\infty) \cup (-1, 0)$$

$$\Rightarrow Df = (1, +\infty) \cup (-1, 0) \quad \text{Ⓟ} \quad \checkmark$$

$$\text{أ) } \frac{r}{x^r + 1} = y \Rightarrow x^r + 1 \neq 0 \Rightarrow x^r \neq -1 \Rightarrow Df = \mathbb{R}$$

$$\text{ب) } \frac{r}{x^r - 1} = y \Rightarrow x^r - 1 \neq 0 \Rightarrow x^r \neq 1 \Rightarrow x \neq 0 \Rightarrow Df = \mathbb{R} - \{0\}$$

$$\text{ج) } \frac{r}{x^r - r} = y \Rightarrow x^r - r \neq 0 \Rightarrow x^r \neq r \Rightarrow x \neq \log_r r \Rightarrow Df = \mathbb{R} - \{\log_r r\} \cup \mathbb{R} - \left\{\frac{1}{r}\right\}$$

$$\text{د) } \frac{r}{x^r - r} = y \Rightarrow x^r - r \neq 0 \Rightarrow x^r \neq r \Rightarrow x \neq \log_r r \Rightarrow Df = \mathbb{R} - \{\log_r r\}$$

Ⓟ ✓

$$\text{أ) } y = (x+1)! \Rightarrow x+1 \in \mathbb{N} \rightarrow x \in \mathbb{N} - 1 \Rightarrow x \in \frac{\mathbb{N}-1}{r} \Rightarrow$$

$$Df = \left\{ x \mid x = \frac{K-1}{r}, K \in \mathbb{N} \right\}$$

Ⓟ ✓

$$\text{ب) } y = \left(\frac{rx - r}{rx - a} \right)! \Rightarrow \frac{rx - r}{rx - a} \in \mathbb{N} \Rightarrow rx - r \in \mathbb{N}(rx - a) - a\mathbb{N} \Rightarrow$$

$$rx - rxw \in r - a\mathbb{N} \Rightarrow x(r - rw) \in r - a\mathbb{N} \Rightarrow x \in \frac{r - a\mathbb{N}}{r - rw}$$

$$\Rightarrow Df = \left\{ x \mid x = \frac{r - aK}{r - rK}, K \in \mathbb{N} \right\}$$